A Multi-Attribute Group Decision Approach Based on Rough Set Theory and Application in Supply Chain Partner Selection

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ABSTRACT
In multi-attribute group decision, decision makers (DMs) are willing or able to provide only incomplete information because of time pressure, lack of knowledge or data, and their limited expertise related with problem domain, so the alternative sets judged by different decision makers are inconsistent in allusion to a certain decision problem, how to form consistent alternative sets becomes a very important problem. There have been a few studies considering incomplete information in group settings, but few papers consider the adjustment of inconsistent alternative sets. We suggest a method, utilizing individual decision results to form consistent alternative sets based on Rough Set theory. The method can be depicted as follows: (1) decision matrix of every decision maker is transformed to decision table through a new discretization algorithm of condition attributes; (2) we analyze the harmony of decision table of every DM in order to filter some extra alternatives with the result that new alternative sets are formed; (3) if the new alternative sets of different DMs are inconsistent all the same, learning quality of DMs for any inconsistent alternative is a standard of accepting the alternative.

Keywords: group decision; inconsistent; Rough Set; reasoning learning

1. INTRODUCTION
A supply chain is a set of facilities, supplies, customers, products and methods of controlling inventory, purchasing, and distribution. As the global economy has become a reality, only through alliances can firms create more value, it becomes very important for studying the relation of alliances, the management of which is a complicated process in which appropriate partner selection is an important phase. Commonly, decision makers from stock, quality, production, technology and R&D department select partners from many partners with whom they expect to cooperate, some difficult problems exist in partner selection: (1) decision makers from different branches have different preference for the measurement indexes and inaccurate information because of time pressure, lack of knowledge and data, so the partner (alternative) sets offered by different decision-makers will be inconsistent 

\[
A_i \cap A_j = \phi
\]

2. THE CAUSE OF THE INCONSISTENT ALTERNATIVE SETS
The process of supply chain partner selection includes filtering wildly, filtering carefully, fining and affirming, tracking and appraising. At the stage of filtering wildly, decision-makers from different branches who have different preference for the measurement indexes often make selections from many alternatives offered, and it is difficult for them to gain complete information, so the alternative sets given by different decision-makers are possibly inconsistent.

With the hypothesis that the alternative definition is same and decision-makers are \(\{DM_i, i = 1, 2, \ldots, m\}\) whose alternatives are \(A_i = \{A_{i1}, i = 1, 2, \ldots, m\}\), for any \(A_i\) and \(A_j\), we can describe their relation as follows (\(\alpha\) is presented in advance):

\[
A_i \cap A_j = \phi
\]
Our aim is to form consistent alternative sets in favor of group appraisal. This paper applies rough set theory, which is introduced by Pawlak (1982) to analyze the inconsistent problem in supply chain partner selection and tries to give a method of forming consistent alternative sets, the adjusting process can be depicted as figure 1:

![Figure 1: Adjusting procedure figure of inconsistent alternatives](image)

### 3. ATTRIBUTES DISCRETIZATION

The process of converting data sets with continuous attributes into input data sets with discrete attributes, called discretization, was studied in many papers [12-16], the major methods include the equal-interval-width method, the equal-frequency method, NaiveScaler method, SemiNaiveScaler method, etc., but these methods don’t contact multi-attribute decision with Rough Set Theory. In the process of multi-attribute decision matrix transformed to decision table, two difficult problems are put forward that are the unknown value of decision attribute and how to discretize the attributes.

In decision table, the significance of condition attributes reflects weight of condition attributes, by whose difference we can discretize the condition attributes. The procedure can be depicted as follows:

1. Because the value of decision attribute \( U(a_p) \) can be computed by many methods, the paper applies TOPSIS method to compute \( U(a_p) \) which reflects the near degree between the value of alternative \( a_p \) and perfect alternative, the decision matrix and patulous matrix of \( DM_i \) (\( i = 1, 2, \ldots, m \)) can be depicted as table 1 and table 2. We can classify \( U(a_p) \) into three sorts: \( U(a_p) > 0.5; U(a_p) < 0.5; U(a_p) = 0.5 \), the coding are 0, 1, 2 respectively.

2. The weight of condition attributes of every decision matrix are computed by entropy method, so we can gain

\[
\ln l + \sum_{i=1}^{n} \ln b_{pi}(i) \ln b_{pi}(i) = \frac{\ln l + \sum_{i=1}^{n} \ln b_{pi}(i) \ln b_{pi}(i)}{\sum_{i=1}^{n} \ln l + \sum_{i=1}^{n} \ln b_{pi}(i) \ln b_{pi}(i)},
\]

the vector of condition attributes judged by \( DM_i \) is \( w(i) = (w_1(i), w_2(i), \ldots, w_n(i)) \).

Table 1: The decision matrix judged by \( DM_i \)

<table>
<thead>
<tr>
<th>( C_i )</th>
<th>( C_2 )</th>
<th>( C_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_{11} )</td>
<td>( b_{11}(i) )</td>
<td>( b_{12}(i) )</td>
</tr>
<tr>
<td>( a_{12} )</td>
<td>( b_{13}(i) )</td>
<td>( b_{14}(i) )</td>
</tr>
<tr>
<td>( \cdots )</td>
<td>( \cdots )</td>
<td>( \cdots )</td>
</tr>
<tr>
<td>( a_{1p} )</td>
<td>( b_{1p}(i) )</td>
<td>( b_{1q}(i) )</td>
</tr>
</tbody>
</table>

Table 2: The patulous decision matrix judged by \( DM_i \)

<table>
<thead>
<tr>
<th>( C_1 )</th>
<th>( C_2 )</th>
<th>( \ldots )</th>
<th>( U(a_p) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_{11} )</td>
<td>( b_{21}(i) )</td>
<td>( \ldots )</td>
<td>( U(a_p) )</td>
</tr>
<tr>
<td>( a_{12} )</td>
<td>( b_{22}(i) )</td>
<td>( \ldots )</td>
<td>( U(a_p) )</td>
</tr>
<tr>
<td>( \cdots )</td>
<td>( \cdots )</td>
<td>( \cdots )</td>
<td>( \cdots )</td>
</tr>
<tr>
<td>( a_{1p} )</td>
<td>( b_{2p}(i) )</td>
<td>( \ldots )</td>
<td>( U(a_p) )</td>
</tr>
</tbody>
</table>

Where: \( \{C_1, C_2, \ldots, C_n\} \) is a finite set of condition attributes, \( b_{pq}(i) \) shows the value of alternative \( a_p \) under the condition attribute \( C_q \) (\( q \in \{1, 2, \ldots, n\} \)).

Because the units of different condition attributes may be inconsistent, \( b_{pq}(i) \) need be changed into...
\[ b_{pq}^r (i) = \frac{b_{pq} (i)}{\sqrt{\sum_{r=1}^{m} b_{pq}^2 (i)}}. \]

(3) For condition attribute \( C_j \), \( b_{pq}^r (i) \) can be arrayed sort ascending, we give the former class number \( m_q = 2 \) and apply classification clustering method to classifying the condition attributes into \( m_q \).

(4) We compute the significance of condition attributes of decision table of every decision maker which is \( r_c (D) - r_{c-c_q} (D) \), the standardization of the value of \( r_c (D) - r_{c-c_q} (D) \) is the effect indexes of condition attributes which is \( v_q (i) = \frac{r_c (D) - r_{c-c_q} (D)}{\sqrt{\sum_{r=1}^{m} (r_c (D) - r_{c-c_q} (D))}} \), if the significance of all condition attributes is 0, the effect indexes of every condition attribute should be same, which is \( \frac{1}{n} \), the vector of the effect indexes can be depicted as \( v(i) = (v_1 (i), v_2 (i), \ldots, v_n (i)) \).

(5) The difference between the significance of condition attributes and the weight of condition attributes
\[ d_{pq} (i) = \frac{\sqrt{\sum_{q=1}^{n} (v_q (i) - w_q (i))^2}}{n} \]
Because the interval spot of condition attributes is finite, the least value exists. In order to reduce to the computing times, we can think of \( d_{pq} (i) \leq \beta \) (\( \beta \) is presented in advance) as the last discretization result If the former discretization cannot satisfy the request which is \( d_{pq} (i) \leq \beta \), we can find the maximal sensitive degree \( \max(\frac{\Delta d_{pq} (i)}{\Delta x_y}) \) and change the class of \( C_q \). At last, we also code the condition attributes such as 0, 1, 2, etc, and compute the classifying interval of all condition attributes at the same time.

4. CONFLICTING ALTERNATIVES ANALYSIS IN DECISION TABLE

Decision table which includes many alternatives is a knowledge expression system, a alternative is a decision rule, extra alternatives exist in decision table because of the deficient condition attributes and untrue stylebooks. In table 2, if \( \{ b_{q}^r (i) \} \) and \( \{ b_{q}^r (i) \} \) denote the perfect point and imperfect point, \( U(a_\gamma) \) can be depicted as follows:
\[ U(a_\gamma) = \sqrt{\sum_{q=1}^{n} (b_{pq} (i) - b_q^r (i))^2} \]

The value of some condition attributes may belong to the same interval in which if the value of \( b_{pq}^r (i) \) tend to upper end, the value of \( \sqrt{\sum_{q=1}^{n} (b_{pq} (i) - b_q^r (i))^2} \) will enlarge and the value of \( \sqrt{\sum_{q=1}^{n} (b_{pq} (i) - b_q^r (i))^2} \) will decrease, at last \( U(a_\gamma) \) will be at different sort.

With the hypothesis of the equation that \( A = \cap_{i=1}^{n} A_i \) and \( A_i = A \cup A_{i_j} \), if some \( A_j \cap (\cap_{i=1}^{n} A_i) = \phi \) exists, a method[17] is offered to adjust \( A_i \). We can analyze the place of conflicting alternatives, if they belong to \( A \), none alternatives will be deleted; if they belong to \( A \) and \( A_i \), we will delete the alternative in \( A_i \); if they belong to \( A_i \), we will analyze the conflicting alternatives by the significance of condition attributes and delete the alternative whose major attribute value is smaller.

So all decision makers will form new alternative sets \( A_{\gamma} = \{ A_1, A_2, \ldots, A_n \} \), where \( A = \{ a_1, a_2, \ldots, \} \), \( A_i = A \cup A_i \).

5. REASONING LEARNING

If the alternative sets \( A_i \) judged by \( DM_i \) are inconsistent, the decision makers need reasoning learning which include how to add new learning rules to former decision table and how to compute the learning quality for the new alternative. We can depict the adjusting process as follows:

(1) With the hypothesis of that the inconsistent alternative sets are \( \cup_{i=1}^{n} A_i \), which is \( A = \{ a_1, a_2, \ldots, a_n \} \), and the learning alternative sets chose by \( DM_i \) is \( \cup_{i=1}^{n} A_i - A_i \).
(2) The weight sets of the decision makers are \( \{ \lambda_1, \lambda_2, \ldots, \lambda_n \} \), we need find the learning decision makers. For example, for inconsistent alternative \( a_1 \), the decision makers sets is \( \{ DM_1, \ldots, DM_n \} \), the weight of the \( DM_t \) \( (t = 1, 2, \ldots, m) \) will change as \( \{ \lambda'_1, \lambda'_2, \ldots, \lambda'_n \} \).

where \( \lambda'_i = \frac{\lambda_i}{\sum \lambda_i} \), the learning attributing value of the learning decision makers for \( a_r \) is \( \{ \sum \lambda'_1 b_{1r}, \ldots, \sum \lambda'_n b_{nr} \} \).

(3) \( DM_y (y = x+1, \ldots, m) \) collates the learning attributing value as the classifying interval of \( DM_x \) and gives the corresponding coding, then the new decision rules come into being.

(4) Computing the learning quality. Considering the relation between learning alternative and intrinsic alternatives, if they conflict, the learning quality can be depicted as follows:

\[
k = \frac{\text{card}(\text{pos}_c(D)) - \text{card}(x_{\text{eq}(c)})}{\text{card}(U) + 1}
\]

where \( x_{\text{eq}(c)} \) denotes the equivalence classification including the new example \( x \).

If the example is new, the learning quality is:

\[
k = \frac{\text{card}(\text{pos}_c(D)) + 1}{\text{card}(U) + 1}
\]

So for alternative \( a_r \), the learning quality is \( f(a_r) = \sum \lambda'_i f(a_r) \), if \( f(a_r) > \gamma \) (\( \gamma \) is presented in advance), the alternatives will be accepted, or the alternative will be deleted.

6. CONCLUSION

This paper analyzes the cause of developing inconsistent alternative sets in multi-attribute group decision, and gives a method of forming consistent alternative sets based on Rough set Theory. The method includes three phrases: attribute discretization, conflicting alternatives deleting in decision table and reasoning learning, and it is applied in supply chain partner selection. In the process of reasoning learning, the paper gives a new rule based on the judgment of the other decision makers, so learning decision makers may get the impact of the other decision makers, how to keep the independence in learning process for the learning decision makers is very difficult, the author will continue studying the problem.

REFERENCE


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