# **Quality Management for an E-Commerce Network Under Budget Constraint**

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### Abstract

In general, several types of information data are transmitted through an E-Commerce network simultaneously. Each type of information data is set to one type of commodity. Under the budget constraint, this paper studies the probability that a given amount of multicommodity can be transmitted through an E-Commerce network, where each node and each arc has several possible capacities. We may take this probability as a performance index for this network. Based on the properties of minimal paths, a simple algorithm is proposed to generate all lower boundary points for  $(d^1, d^2, \dots, d^p; C)$  where  $d^i$  is the demand of commodity i and C is the budget. The probability can then be calculated in terms of such points.

### 1. Introduction

The capacity of each arc (the maximum flow passing the arc per unit time) in a binary-state flow network has two levels, 0 and a positive integer. For perfect nodes case, Aggarwal et al. [1] computed the system reliability, the probability that the maximum flow of the network is not less than the demand, in terms of minimal paths (MPs). A MP is an ordered sequence of arcs from the source s to the sink t that has no cycle. Lee et al. [10] and Rueger [15] extended the system reliability problem to the case that nodes and arcs have a positive-integer capacity and may fail. A stochastic-flow network is a multi-state network in which each arc has several states or capacities. The system reliability is the probability that the maximum flow of single-commodity through the network is not less than the demand d. Without the budget constraint, several authors [11,12,14,19,21] had presented algorithms to generate lower boundary points for d in order to evaluate the system reliability for perfect node case. Lin [13] and Yeh [22] extended the problem to the more general case that nodes have several capacities as arcs do.

However, in real world, many stochastic-flow networks allow multicommodity to be transmitted from s to t simultaneously. Assuming the flow network is deterministic (i.e., the capacity of each arc is a constant), many authors [3,4,8,17,18] studied the multicommodity minimum cost flow problem, which is to minimize the total cost of multicommodity. The purpose of this paper is to extend the system reliability problem to a

The Second International Conference on Electronic Business Taipei, Taiwan, December 10-13, 2002 multicommodity case, named multicommodity reliability here, for a stochastic-flow network with node failure under budget constraint. Then a MP is an ordered sequence of arcs and nodes from s to t that has no cycle. The system reliability is the probability that the given demand  $(d^1, d^2, ..., d^p)$  can be transmitted through the stochastic-flow network under budget C, where  $d^k$ , k = 1, 2, ..., p, is the required demand of commodity k. A simple algorithm is proposed to generate all lower boundary points for  $(d^1, d^2, ..., d^p; C)$ , then the multicommodity reliability can be computed in terms of all lower boundary points for  $(d^1, d^2, ..., d^p; C)$ .

# 2. Multicommodity Model Under Budget Constraint

G = (A, N, M) is a stochastic-flow network with source *s* and sink *t* where  $A = \{a_i | 1 \le i \le n\}$  the set of arcs,  $N = \{a_i | n + 1 \le i \le n + r\}$  the set of nodes and  $M = (M_1, M_2, ..., M_{n+r})$  with  $M_i$  the maximal capacity of  $a_i$ . Let  $x_i$ denote the (current) capacity of  $a_i$ , and it takes values from  $\{0, 1, 2, ..., M_i\}$  with a given probability distribution.

# 2.1 Assumptions and Nomenclature

- 1. All commodities are transmitted from *s* to *t*.
- 2. The capacities of different arcs are statistically independent.
- $\lceil x \rceil$  the smallest integer such that  $\lceil x \rceil \ge x$
- $Y \ge X$   $(y_1, y_2, ..., y_{n+r}) \le (x_1, x_2, ..., x_{n+r})$  if and only if  $y_i \ge x_i$  for i = 1, 2, ..., n+r
- Y > X  $(y_1, y_2, ..., y_{n+r}) > (x_1, x_2, ..., x_{n+r})$  if and only if  $Y \ge X$  and  $y_i > x_i$  for at least one *i*

### 2.2 Multicommodity Flow

Suppose  $P_1, P_2, ..., P_m$  are MPs form *s* to *t*. The multicommodity flow model for *G* is described in terms of the capacity vector  $X = (x_1, x_2, ..., x_{n+r})$  and the flow assignment  $(F^1, F^2, ..., F^p)$ , where  $F^k = (f_1^k, f_2^k, ..., f_m^k)$  with  $f_j^k$  denoting the flow (integer-value) of commodity *k* through  $P_j, j = 1, 2, ..., m, k = 1, 2, ..., p$ . Such an  $(F^1, F^2, ..., F^p)$  which is feasible under *X* satisfies the following condition:

$$\left[\sum_{k=1}^{p} (\boldsymbol{\sigma}_{i}^{k} \cdot \sum_{a_{i} \in P_{j}} f_{j}^{k})\right] \leq x_{i} \text{ for } i = 1, 2, ..., n+r, \qquad (1)$$

where  $\overline{\sigma}_i^k$  (real number) is the weight of commodity kon  $a_i$  i.e., the consumed amount of capacity on  $a_i$  per commodity k. For convenience, let  $\phi_X$  denote the set of  $(F^1, F^2, ..., F^p)$  feasible under X. Similarly,  $(F^1, F^2, ..., F^p)$  $\in \phi_M$  if it satisfies

$$\left[\sum_{k=1}^{p} (\boldsymbol{\varpi}_{i}^{k} \cdot \sum_{a_{i} \in P_{j}} f_{j}^{k})\right] \leq M_{i} \text{ for } i = 1, 2, ..., n + r. \quad (2)$$

Let  $c_i^k$  denote the transportation cost of each commodity *k* through  $a_i$ . Under *X*, the network *G* satisfies the given demand  $(d^1, d^2, ..., d^p)$  under the budget *C* if there exists an  $(F^1, F^2, ..., F^p) \in \phi_X$  satisfying constraints (3) and (4);

$$\sum_{j=1}^{m} f_{j}^{k} = d^{k}, k = 1, 2, ..., p$$
(3)

$$\sum_{i=1}^{n+r} \sum_{k=1}^{p} \left( c_i^k \cdot \sum_{a_i \in mp_j} f_j^k \right) \leq C.$$
(4)

Let  $\Omega = \{X | \text{ there exists an } (F^1, F^2, \dots, F^p) \in \phi_X \text{ satisfying constraints (3) and (4)} \}$ . The multicommodity reliability  $R_{d^1, d^2, \dots, d^p; C}$  is thus

$$R_{d^1,d^2,\ldots,d^p;C} = \Pr\{\Omega\} = \sum_{X \in \Omega} \Pr\{X\}$$

Each minimal one in  $\Omega$  is named a lower boundary point for  $(d^1, d^2, \dots, d^p; C)$  throughout this paper i.e., X is a lower boundary point for  $(d^1, d^2, \dots, d^p; C)$  if and only if i)  $X \in \Omega$ and ii)  $Y \notin \Omega$  for any capacity vector Y such that Y < X. Hence,

 $R_{d^1,d^2,\ldots,d^p;C} = \Pr\{Y|Y \ge X \text{ for a lower boundary point for} (d^1,d^2,\ldots,d^p;C) X\}.$ 

# 2.3 Generate All Lower Boundary Points for $(d^1, d^2, ..., d^p; C)$

Let  $\Phi = \{(F^1, F^2, ..., F^p) | (F^1, F^2, ..., F^p) \text{ satisfies}$ constraints (2) - (4)}. For each  $(F^1, F^2, ..., F^p) \in \Phi$ , generate the capacity vector  $Z_{F^1, F^2, ..., F^p} = (z_1, z_2, ..., z_{n+r})$ via

$$z_i = \left[\sum_{k=1}^p (\boldsymbol{\varpi}_i^k \cdot \sum_{a_i \in P_j} f_j^k)\right] \text{ for } i = 1, 2, ..., n+r. \quad (5)$$

For convenience, let  $\Psi = \{ Z_{F^1, F^2, \dots, F^p} | (F^1, F^2, \dots, F^p) \in \Phi \}.$ We will first see that  $\Psi$  contains all lower boundary.

We will first see that  $\Psi$  contains all lower boundary points for  $(d^1, d^2, \dots, d^p; C)$  in the following lemma.

**Lemma 1.** Let X be a lower boundary point for  $(d^1, d^2, ..., d^p; C)$ . Then  $X = Z_{F^1, F^2, ..., F^p}$  for each  $(F^1, F^2, ..., F^p) \in \phi_X \cap \Phi$ . **Proof:** For each  $(F^1, F^2, ..., F^p) \in \phi_X \cap \Phi$ , constraint (1) says that  $Z_{F^1, F^2, ..., F^p} \leq X$ . Suppose that  $Z_{F^1, F^2, ..., F^p} \leq X$ , then  $Z_{F^1, F^2, ..., F^p} \notin \Omega$  as X is minimal in  $\Omega$ . This is a

contradiction. Hence,  $X = Z_{F^1, F^2, \dots, F^p}$ .

The following lemma further shows that  $\Psi_{\min} \equiv \{X|X \text{ is minimal in } \Psi\}$  is the set of lower boundary point for  $(d^1, d^2, \dots, d^p; C)$ .

**Lemma 2.** {X|X is a lower boundary point for  $(d^1, d^2, ..., d^p; C)$ } =  $\Psi_{\min}$ .

**Proof:** Firstly, suppose that X is a lower boundary point for  $(d^1, d^2, ..., d^p; C)$  (note that  $X \in \Psi$  by lemma 1) but  $X \notin \Psi_{\min}$  i.e., there exist a  $Y \in \Psi$  such that Y < X. Then  $Y \in \Omega$ , which contradicts to that X is a lower boundary point for  $(d^1, d^2, ..., d^p; C)$ . Hence,  $X \in \Psi_{\min}$ .

Conversely, suppose that  $X \in \Psi_{\min}$  (note that  $X \in \Omega$ ) but it is not a lower boundary point for  $(d^1, d^2, \dots, d^p; C)$ . Then there exists a lower boundary point for  $(d^1, d^2, \dots, d^p; C)$  *Y* s.t. Y < X. By lemma 1,  $Y \in \Psi$  that contradicts to that  $X \in \Psi_{\min}$ . Hence, *X* is a lower boundary point for  $(d^1, d^2, \dots, d^p; C)$ .

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# 3. Algorithm

As those approaches of [11,13,14,19,21,22] we suppose all MPs have been pre-computed. Minimal paths can be efficiently derived from those algorithms discussed in [2,9,16]. The algorithm of Al-Ghanim [2] showed an approximate linear time response versus the number of network nodes. Kobayashi and Yamamoto [9] showed that to generate all minimal paths for a random network with 30 nodes and 100 arcs takes no more than 1300 seconds.

**Step 1.** Obtain all  $(F^1, F^2, ..., F^p)$  with  $F^k = (f_1^k, f_2^k, ..., f_m^k), k = 1, 2, ..., p$ , of the following constraints:

$$\left|\sum_{k=1}^{p} \left( \overline{\sigma}_{i}^{k} \cdot \sum_{a_{i} \in P_{j}} f_{j}^{k} \right) \right| \leq M_{i} \quad \text{for } i = 1, 2, ..., n + r(5)$$

$$\sum_{j=1}^{m} f_{j}^{k} = d^{k}, \, k = 1, 2, ..., p$$
(6)

$$\sum_{i=1}^{n+r} \sum_{k=1}^{p} \left( c_i^k \cdot \sum_{a_i \in mp_j} f_j^k \right) \le C$$
(7)

**Step 2.** Transform each  $(F^1, F^2, ..., F^p)$  into  $X = (x_1, x_2, ..., x_{n+r})$  according to

$$x_i = \left[\sum_{k=1}^p \left(\boldsymbol{\varpi}_i^k \cdot \sum_{a_i \in P_j} f_j^k\right)\right] \text{ for } i = 1, 2, ..., n+r \quad (8)$$

**Step 3.** Suppose  $\Psi = \{X_1, X_2, ..., X_{\nu}\}.$ 

- 3.1)  $I = \phi(I \text{ is the stack which stores the index})$ of each non-minimal X after checking. Initially,  $I = \phi$ .)
- 3.2) For i = 1 To v and  $i \notin I$
- 3.3) For j = i + 1 To v with  $j \notin I$
- 3.4) If  $X_i \ge X_j$ ,  $I = I \cup \{i\}$  and go to step 3.7) Elseif  $X_j > X_i$ ,  $I = I \cup \{j\}$
- 3.5) j = j + 1
- 3.6)  $X_i$  is a lower boundary point for  $(d^1, d^2, \dots, d^p; C)$
- 3.7) i = i + 1
- 3.8) End.

# 4. A numerical example



We use the benchmark [14,20] in Figure 1 to illustrate the proposed approach. There are 7 MPs:  $P_1 =$  $\{a_{13}, a_1, a_9, a_3, a_{11}, a_7, a_{14}\}, P_2 = \{a_{13}, a_1, a_9, a_3, a_{11}, a_6, a_{11}, a_{11}, a_{12}, a_{13}, a_{$  $(a_{12}, a_8, a_{14}), P_3 = \{a_{13}, a_1, a_9, a_4, a_{12}, a_8, a_{14}\}, P_4 = \{a_{13}, a_{12}, a_{13}, a_{12}, a_{13}, a_{14}\}$  $a_1, a_9, a_4, a_{12}, a_6, a_{11}, a_7, a_{14}$ ,  $P_5 = \{a_{13}, a_2, a_{10}, a_5, a_{12}, a_8, a_{13}, a_{14}, a_{$  $a_{14}$ ,  $P_6 = \{a_{13}, a_2, a_{10}, a_5, a_{12}, a_6, a_{11}, a_7, a_{14}\}$  and  $P_7 =$  $\{a_{13}, a_2, a_{10}, a_5, a_{12}, a_4, a_9, a_3, a_{11}, a_7, a_{14}\}$ . The data of arcs and nodes for 2-commodity case are shown in Table 1. We assume the source and the sink both have infinite capacity and are perfect. If the demand  $(d_1, d_2)$  is set to be (3,3) and C = 810 US dollars, then the multicommodity reliability  $R_{3,3;810}$  can be calculated by the following steps. **Step 1.** Obtain all  $F^1 = (f_1^1, f_2^1, f_3^1, f_4^1, f_5^1, f_6^1, f_7^1)$ , and  $F^2 = (f_1^2, f_2^2, f_3^2, f_4^2, f_5^2, f_6^2, f_7^2)$  of the following integer-programming: (9)  $a_1: \int f_1^1 + f_2^1 + f_3^1 + f_4^1 + 2f_1^2 + 2f_2^2 +$  $2f_3^2 + 2f_4^2 \rceil \le 5$  $a_{2}:\left[f_{5}^{1}+f_{6}^{1}+f_{7}^{1}+2f_{5}^{2}+2f_{6}^{2}+2f_{7}^{2}\right] \leq 5$  $a_3: \left[ f_1^1 + f_2^1 + f_2^1 + 2 f_1^2 + 2 f_2^2 + 2 f_2^2 \right] \le 5$ 

$$a_{4}: \left\lceil f_{3}^{1} + f_{4}^{1} + f_{7}^{1} + 2f_{3}^{2} + 2f_{4}^{2} + 2f_{7}^{2} \right\rceil \le 5$$
  
$$a_{5}: \left\lceil f_{5}^{1} + f_{6}^{1} + f_{7}^{1} + 2f_{5}^{2} + 2f_{6}^{2} + 2f_{7}^{2} \right\rceil \le 5$$
  
$$a_{6}: \left\lceil f_{2}^{1} + f_{4}^{1} + f_{6}^{1} + 2f_{2}^{2} + 2f_{4}^{2} + 2f_{6}^{2} \right\rceil \le 5$$

$$\begin{aligned} a_{7}: \left| f_{1}^{1} + f_{4}^{1} + f_{6}^{1} + f_{7}^{1} + 2f_{1}^{2} + 2f_{4}^{2} + 2f_{6}^{2} + 2f_{7}^{2} \right| &\leq 5 \\ a_{8}: \left\lceil f_{2}^{1} + f_{3}^{1} + f_{5}^{1} + 2f_{2}^{2} + 2f_{3}^{2} + 2f_{5}^{2} \right\rceil &\leq 5 \\ a_{9}: \left\lceil f_{1}^{1} + f_{2}^{1} + f_{3}^{1} + f_{4}^{1} + f_{7}^{1} + 2f_{1}^{2} + 2f_{2}^{2} + 2f_{3}^{2} + 2f_{4}^{2} + 2f_{7}^{2} \right\rceil &\leq 9 \\ a_{10}: \left\lceil f_{5}^{1} + f_{6}^{1} + f_{7}^{1} + 2f_{5}^{2} + 2f_{6}^{2} + 2f_{7}^{2} \right\rceil &\leq 9 \\ a_{11}: \left\lceil f_{1}^{1} + f_{2}^{1} + f_{4}^{1} + f_{6}^{1} + f_{7}^{1} + 2f_{1}^{2} + 2f_{2}^{2} + 2f_{4}^{2} + 2f_{6}^{2} + 2f_{7}^{2} \right\rceil &\leq 9 \\ a_{12}: \left\lceil f_{1}^{2} + f_{3}^{1} + \dots + f_{7}^{1} + 2f_{2}^{2} + 2f_{3}^{2} + \dots + 2f_{7}^{2} \right\rceil &\leq 9 \end{aligned}$$

(10) 
$$f_1^1 + f_2^1 + f_3^1 + f_4^1 + f_5^1 + f_6^1 + f_7^1 = 3$$
  
 $f_1^2 + f_2^2 + f_3^2 + f_4^2 + f_5^2 + f_6^2 + f_7^2 = 3$ 

$$(11) \{ 10(f_1^1 + f_2^1 + f_3^1 + f_4^1) + \\ 20(f_1^2 + f_2^2 + f_3^2 + f_4^2) \} + \\ \{ 20(f_5^1 + f_6^1 + f_7^1) + 30(f_5^2 + f_6^2 + f_7^2) \} + \\ \{ 30(f_1^1 + f_2^1 + f_7^1) + 40(f_1^2 + f_2^2 + f_7^2) \} + \\ \{ 20(f_3^1 + f_4^1 + f_7^1) + 40(f_3^2 + f_4^2 + f_7^2) \} + \\ \{ 20(f_3^1 + f_6^1 + f_7^1) + 20(f_5^2 + f_6^2 + f_7^2) \} + \\ \{ 20(f_2^1 + f_4^1 + f_6^1) + 30(f_2^2 + f_4^2 + f_6^2) \} + \\ \{ 20(f_1^2 + f_4^2 + f_6^2 + f_7^2) \} + \\ \{ 20(f_1^1 + f_4^1 + f_6^1 + f_7^1) + \\ 40(f_1^2 + f_4^2 + f_6^2 + f_7^2) \} + \\ \{ 20(f_1^1 + f_2^1 + f_3^1 + f_4^1 + f_7^1) + \\ 30(f_1^2 + f_2^2 + f_3^2 + f_4^2 + f_7^2) \} + \\ \{ 20(f_1^1 + f_2^1 + f_3^1 + f_4^1 + f_7^1) + \\ 30(f_1^2 + f_2^2 + f_3^2 + f_4^2 + f_7^2) \} + \\ \{ 20(f_1^1 + f_2^1 + f_4^1 + f_6^1 + f_7^1) + \\ 30(f_1^2 + f_2^2 + f_3^2 + f_4^2 + f_7^2) \} + \\ \{ 20(f_2^1 + f_3^1 + ... + f_7^1) + \\ 30(f_2^2 + f_3^2 + ... + f_7^2) \} \\ \leq 810$$

Seven  $(F^1, F^2)$  are obtained: (3, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 2, 0, 0), (2, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 2, 0, 0), (2, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 2, 0, 0), (2, 0, 0, 0, 1, 0, 0), (0, 0, 1, 0, 2, 0, 0), (1, 0, 0, 0, 2, 0, 0, 2, 0, 0, 0, 1, 0, 0), (0, 0, 1, 0, 2, 0, 0, 2, 0, 0, 0, 1, 0, 0), (0, 0, 0, 1, 0, 2), (0, 0, 1, 0, 0) and (0, 0, 0, 0, 2, 1, 0, 2, 0, 0, 0, 1, 0, 0). And the corresponding costs are 790, 790, 780, 780, 780, 770 and 810, respectively.

**Step 2.** Transform all  $(F^1, F^2)$  into  $X = (x_1, x_2, ..., x_{12})$  according to

$$x_{1} = \left| \begin{array}{c} f_{1}^{1} + f_{2}^{1} + f_{3}^{1} + f_{4}^{1} + 2f_{1}^{2} + 2f_{2}^{2} + \\ 2f_{3}^{2} + 2f_{4}^{2} \end{array} \right|$$

$$\begin{aligned} x_{2} &= \left\lceil f_{5}^{1} + f_{6}^{1} + f_{7}^{1} + 2f_{5}^{2} + 2f_{6}^{2} + 2f_{7}^{2} \right\rceil \\ x_{3} &= \left\lceil f_{1}^{1} + f_{2}^{1} + f_{7}^{1} + 2f_{1}^{2} + 2f_{2}^{2} + 2f_{7}^{2} \right\rceil \\ x_{4} &= \left\lceil f_{3}^{1} + f_{4}^{1} + f_{7}^{1} + 2f_{3}^{2} + 2f_{4}^{2} + 2f_{7}^{2} \right\rceil \\ x_{5} &= \left\lceil f_{5}^{1} + f_{6}^{1} + f_{7}^{1} + 2f_{5}^{2} + 2f_{6}^{2} + 2f_{7}^{2} \right\rceil \\ x_{6} &= \left\lceil f_{2}^{1} + f_{4}^{1} + f_{6}^{1} + 2f_{2}^{2} + 2f_{4}^{2} + 2f_{6}^{2} \right\rceil (12) \\ x_{7} &= \left\lceil f_{1}^{1} + f_{4}^{1} + f_{6}^{1} + f_{7}^{1} + 2f_{1}^{2} + 2f_{4}^{2} + 2f_{6}^{2} \right\rceil \\ x_{8} &= \left\lceil f_{2}^{1} + f_{3}^{1} + f_{5}^{1} + 2f_{2}^{2} + 2f_{3}^{2} + 2f_{5}^{2} \right\rceil \\ x_{9} &= \left\lceil f_{1}^{1} + f_{2}^{1} + f_{3}^{1} + f_{4}^{1} + f_{7}^{1} + 2f_{1}^{2} + 2f_{7}^{2} \right\rceil \\ x_{10} &= \left\lceil f_{5}^{1} + f_{6}^{1} + f_{7}^{1} + 2f_{5}^{2} + 2f_{6}^{2} + 2f_{7}^{2} \right\rceil \\ x_{11} &= \left\lceil f_{1}^{1} + f_{2}^{1} + f_{4}^{1} + f_{6}^{1} + f_{7}^{1} + 2f_{1}^{2} + 2f_{2}^{2} + 2f_{4}^{2} + 2f_{7}^{2} \right\rceil \\ x_{12} &= \left\lceil f_{2}^{1} + f_{3}^{1} + \dots + f_{7}^{1} + 2f_{2}^{2} + 2f_{3}^{2} + \dots + 2f_{7}^{2} \right\rceil \end{aligned}$$

We obtain  $X_1 = (5, 4, 5, 0, 4, 0, 5, 4, 5, 4, 5, 4), X_2 = (5, 4, 4, 1, 4, 0, 4, 5, 5, 4, 4, 5), X_3 = (4, 5, 4, 0, 5, 0, 4, 5, 4, 5, 4, 5), X_4 = (5, 4, 5, 0, 4, 0, 5, 4, 5, 4, 5, 4), X_5 = (5, 4, 4, 1, 4, 0, 4, 5, 5, 4, 4, 5), X_6 = (4, 5, 4, 0, 5, 0, 4, 5, 4, 5, 4, 5)$ and  $X_7 = (4, 5, 4, 0, 5, 1, 5, 4, 4, 5, 5, 5).$ 

**Step 3.** Check each  $X_i$  whether it is a lower boundary point for (3,3;810) or not.

 $I = \phi$ 3.1) *i* = 1 3.2) 3.3) j = 2 $X_1 \not\geq X_2$  and  $X_2 \not\geq X_1$ .  $I = \{\phi\}$ . 3.4) *j* = 3 3.3)  $X_1 \not\geq X_3$  and  $X_3 \not\geq X_1$ .  $I = \{\phi\}$ . 3.4) 3.5) j = 43.4)  $X_1 \ge X_4$ .  $I = \{1\}$ . i = 23.2) :

After further checking,  $X_4$ ,  $X_5$ ,  $X_6$  and  $X_7$  are all lower boundary points for (3,3;810). Let  $B_1 = \{X|X \ge X_4\}$ ,  $B_2 = \{X|X \ge X_5\}$ ,  $B_3 = \{X|X \ge X_6\}$  and  $B_4 = \{X|X \ge X_7\}$ . Hence, the multicommodity reliability  $R_{3,3;810} = \Pr\{B_1 \cup B_2 \cup B_3 \cup B_4\} = 0.61623376$  can be computed by the inclusion-exclusion method.

# 5. Conclusions

This article extends the system reliability problem to the multicommodity reliability for a stochastic-flow network with node failure under budget constraint. The multicommodity reliability is the probability that the demand  $(d^1, d^2, ..., d^p)$  can be transmitted through the stochastic-flow network under budget *C*. Based on the properties of minimal paths, we propose a simple algorithm to generate all lower boundary point for  $(d^1, d^2, ..., d^p; C)$ . Then the multicommodity reliability can be calculated in terms of lower boundary points for  $(d^1, d^2, \dots, d^p; C)$  by applying the inclusion-exclusion method. In our model the transportation cost  $c_i^k$  is not assumed to be linear in  $\varpi_i^k$ . The main reason is that the transportation cost is not only dependent on the dimension of commodity but also on other attributes of commodity. For example, poison, vulnerable, fragile, etc. For the case that the transportation cost is only charged in terms of consumed capacity,  $c_i^k$  is linear in  $\varpi_i^k$ . However, this condition is a special case of the proposed model.

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Table 1. The data of arcs and nodes for 2-commodity example

		2-001	innou	ILY EX	ample	
Arc	Capacity	Probability	$\sigma_{i}^{1}$	$\sigma_i^2$	$c_i^1$	$c_i^2$
	1 2	2			(US	(US
					dollar)	dollar)
$a_1$	0*	.01	1	2	10	20
1	1	01	-	_		
	2	01				
	3	02				
	4	02				
	5	93				
<i>a</i> <sub>2</sub>	0	01	1	2	20	30
u2	1	01	1	4	20	50
	2	.01				
	2	.02				
	5 4	.03				
	5	90				
а.	0	.90	1	2	30	40
u <sub>3</sub>	1	.01	1	2	50	40
	2	.01				
	2	.01				
	5	.02				
	5	.02				
a	5	.95	1	r	20	40
$u_4$	1	.01	1	2	20	40
	1	.01				
	2	.02				
	5	.03				
	4	.03				
-	5	.90	1	n	10	20
$a_5$	0	.01	1	2	10	20
	1	.01				
	2	.01				
	3	.02				
	4	.02				
	2	.93	1	2	20	20
$a_6$	0	.01	I	2	20	30
	1	.01				
	2	.02				
	3	.03				
	4	.03				
	2	.90	1	2	20	10
$a_7$	0	.01	I	2	30	40
	1	.01				
	2	.01				
	3	.02				
	4	.02				
	5	.93		•	•	10
$a_8$	0	.01	I	2	20	40
	1	.01				
	2	.02				
	3	.03				
	4	.03				
	5	.90		-	• •	•
$a_9 \sim$	0	.01	1	2	20	30
$a_{12}$	1	.01				
	3	.01				
	5	.01				
	7	.02				
	9	.94				

\*Pr{the capacity of  $a_1$  is 0} = 0.01.