

Upstream R&D Competition and Cooperation in a Two-Tier Supply Chain

Pei-Jun Huo, Jian Chen

School of Economics and Management

Tsinghua University

Beijing 100084, China

{huopj, chenj}@em.tsinghua.edu.cn

Ji-Xiang Chen

Department of Business Administration

Shanghai Jiao Tong University

Shanghai 200052, China

jixichen@fm365.com

Abstract

This paper derives a two-tier supply chain model with many firms in each tier. The upstream firms engage in cost-reducing R&D activities. Under the case of R&D competition, the paper discusses how the changes in R&D spillover, R&D efficiency and the numbers of the manufacturers and the suppliers affect the R&D expenditures, the quantities and the profits. Then, R&D cooperation is considered. And what effects arise with the changes in the R&D spillover and the degree of cooperation are studied.

Key words: supply chain, upstream firm, downstream firm, R&D competition, R&D cooperation

1. Introduction

The subject of R&D competition and cooperation has gained a lot of attention from researchers in a variety of settings. Such literature usually uses a two-stage game that firms choose their R&D expenditures before making decisions on production. [1] examines the impact of multinational presence on domestic firms' innovative efforts in a model focusing on the strategic dimension of R&D. [2] addresses the question about the optimal degree of spillovers and the number of rival firms necessary for obtaining the maximum amount of effective R&D. Both [1] and [2] only consider R&D competition between firms. Meanwhile, many papers consider R&D competition and cooperation simultaneously and make comparisons between them. [3] presents an interesting analysis of cooperative and non-cooperative R&D and compares the R&D investment and output under cooperative R&D with those under non-cooperative R&D. [4] extends the results in [3] to the case of more than two firms and more general demand and cost assumptions. [5]

extends the analytical framework in [3] to a two-industry, two-firm-per-industry model allowing for R&D spillovers to occur within industries as well as between industries. [6] compares duopoly outcomes between two alternative models of independent R&D and non-cooperative RJVs, where there are complementarities between firm-specific R&D resources.

Besides the two-stage models, some papers introduce another stage game before the other games occur. [7] derives the non-cooperative, optimal policy towards international R&D cooperation. In the model, the governments simultaneously announce their R&D subsidy rates before the firms choose their R&D expenditures and quantities. [8] examines the impact of the firms' mode of foreign expansion on the incentive to innovate as well as the effects of R&D activities and technological spillovers on the firms' international strategy. [8] considers a two country imperfect competition model where the firms face three different type of decisions: how to expand abroad, how much to spend on R&D and how much to sell in each market.

Although there exist a lot of papers about R&D competition and cooperation, we can hardly find one under the case of supply chain. An exception may be [9] which derives a model that an integrated firm produces the input and engages in R&D to reduce the input production cost. The integrated firm sells the input to its rival at a regulated price and competes with it in the final product market. [9] examines input price regulation's effects on R&D and output. In our paper, we consider a two-tier supply chain with many firms in each tier. The upstream firms, the suppliers, conduct R&D activities that result in reduction in marginal cost. We use a parameter β to capture R&D spillovers between the

suppliers.¹ We first consider R&D competition and then R&D cooperation. Concerning R&D competition, we discuss how the changes in R&D spillover, R&D efficiency and the numbers of the manufacturers and the suppliers affect the R&D expenditures, the quantities and the profits. Under R&D cooperation, we study what effects arise with the changes in the R&D spillover and the degree of cooperation.

The two-tier-supply-chain structure with many firms in each tier in the paper is similar to that of [10]. Such kind of structure also appears in [11] and [12]. [10] examines the impact of fixed and variable costs on the structure and competitiveness. [11] examines vertical integration as an equilibrium phenomenon and consider the issue of private profitability versus collective profitability. [12] analyses the effects of different institutional arrangements of union-firm bargaining.

2. The Model

Consider a two-tier supply chain with n_1 firms, the manufacturers, in the downstream tier, and n_2 firms, the suppliers, in the upstream tier. The firms in the same tier engage in Cournot competition. The suppliers sell a homogeneous input at price p_2 to the manufacturers which use it to produce a final product. Without loss of generality, we assume that one unit of final product requires one unit of input. The inverse demand function of the final product is $p_1 = a - bQ_1$, where p_1 is the price and Q_1 is the total output. Denote the output of manufacturer i as $q_{1,i}$, then $Q_1 = \sum_{i=1}^{n_1} q_{1,i}$. The profit of a representative manufacturer is

$$\Pi_{1,i} = (p_1 - v_1 - p_2)q_{1,i} \quad (1)$$

where v_1 is the constant marginal cost of the final

product.

The suppliers conduct R&D activities that result in reduction in marginal cost. Let x_j be the level of R&D investment undertaken by supplier j and let $v_{2,j}$ denote supplier j 's marginal cost. In order to model the possibility of imperfect appropriability (i.e., technological spillovers between the suppliers), we introduce a spillover parameter $\beta \in [0, 1]$. This means that the magnitude of supplier j 's cost reduction is determined by its own technological knowledge and by a fraction β of the sum of the other suppliers' knowledge. More specifically

$$v_{2,j} = v_2^0 - \sqrt{gx_j} - \beta \sum_{\substack{m=1 \\ m \neq j}}^{n_2} \sqrt{gx_m} \quad (2)$$

where the parameters g and v_2^0 describe the efficiency of the R&D process and the initial marginal production cost of the suppliers. The expression $\sqrt{gx_j}$ is an R&D production function which reflects the existence of diminishing return to R&D expenditures and can be seen in many papers^{[3][8][13][14]}. The profit of supplier j is

$$\Pi_{2,j} = (p_2 - v_{2,j})q_{2,j} - x_j \quad (3)$$

where $q_{2,j}$ is supplier j 's output.

Now, we consider the decision problem of the manufacturers. Manufacturer i chooses its output to maximize its profit by taking the other manufacturers' output as given. The first-order condition can be derived from (1), which is

$$a - v_1 - p_2 - b(Q_1 + q_{1,i}) = 0$$

Since the manufacturers are identical, we can get

$$q_{1,i} = q_1 = \frac{a - v_1 - p_2}{b(n_1 + 1)}$$

and

¹ The spillovers are often important parameters in the R&D literature and almost all the aforementioned papers consider the spillovers. [9] ignores the spillovers since only one firm conducts R&D activity and it is no need to consider them. It should also be noted that the spillover in [7] has the different meaning from other papers and ours. [7] uses the spillover effect to capture the degree of R&D collaboration while such parameters in other papers indicates the involuntary technological information leakage.

$$Q_1 = n_1 q_1 = \frac{n_1(a - v_1 - p_2)}{b(n_1 + 1)} \quad (4)$$

In equilibrium, the overall output of the manufacturers and the suppliers must be equal. Substituting Q_2 for Q_1 , (4) can be rearranged to

$$p_2 = (a - v_1) - \frac{b(n_1 + 1)}{n_1} Q_2 \quad (5)$$

where $Q_2 = \sum_{m=1}^{n_2} q_{2,m}$ is the overall output of the suppliers. Supplier j chooses $q_{2,j}$ to maximize its profit.

From (3) and (5) we get the first-order condition as follows

$$a - v_1 - v_{2,j} - \frac{b(n_1 + 1)}{n_1} (Q_2 + q_{2,j}) = 0 \quad (6)$$

We can get n_2 equations from (6) since j varies from 1 to n_2 . Then, we can, respectively, derive the suppliers' total output and supplier j 's output as follows

$$Q_2 = \frac{n_1}{b(n_1 + 1)(n_2 + 1)} [n_2(a - v_1) - \sum_{m=1}^{n_2} v_{2,m}] \quad (7)$$

$$q_{2,j} = \frac{n_1}{b(n_1 + 1)(n_2 + 1)} [(a - v_1) - (n_2 + 1)v_{2,j} + \sum_{m=1}^{n_2} v_{2,m}] \quad (8)$$

Substituting (5), (7) and (8) into (3), we get

$$\Pi_{2,j} = \frac{n_1}{b(n_1 + 1)(n_2 + 1)^2} [(a - v_1) - (n_2 + 1)v_{2,j} + \sum_{m=1}^{n_2} v_{2,m}]^2 - x_j \quad (9)$$

3. R&D Competition

In this section, we first derive the equilibrium when the suppliers independently decide R&D expenditures to maximize their individual profits. Then we discuss what effects arise with changes in some parameters.

Substituting (2) into (9) results in

$$\Pi_{2,j} = \frac{n_1}{b(n_1 + 1)(n_2 + 1)^2} [(a - v_1 - v_2^0) + (n_2 + \beta - n_2\beta)\sqrt{gx_j} + (2\beta - 1)\sum_{\substack{m=1 \\ m \neq j}}^{n_2} \sqrt{gx_m}]^2 - x_j \quad (10)$$

Supplier j chooses its R&D investment, x_j , to maximize its profit. The first-order condition is

$$\frac{\partial \Pi_{2,j}}{\partial x_j} = \frac{n_1}{b(n_1 + 1)(n_2 + 1)^2} [(a - v_1 - v_2^0) + (n_2 + \beta - n_2\beta)\sqrt{gx_j} + (2\beta - 1)\sum_{\substack{m=1 \\ m \neq j}}^{n_2} \sqrt{gx_m}] \frac{(n_2 + \beta - n_2\beta)g}{\sqrt{gx_j}} - 1 = 0$$

from which we get

$$\sqrt{gx_j} = \sqrt{gx} = \frac{n_1 g(n_2 + \beta - n_2\beta)(a - v_1 - v_2^0)}{b(n_1 + 1)(n_2 + 1)^2 - n_1 g(1 - \beta + n_2\beta)(n_2 + \beta - n_2\beta)}$$

or

$$x_j = x = \frac{n_1^2 g(n_2 + \beta - n_2\beta)^2 (a - v_1 - v_2^0)^2}{[b(n_1 + 1)(n_2 + 1)^2 - n_1 g(1 - \beta + n_2\beta)(n_2 + \beta - n_2\beta)]^2} \quad (11)$$

We need the following condition

$$b(n_1 + 1)(n_2 + 1)^2 > n_1 g(1 - \beta + n_2\beta)(n_2 + \beta - n_2\beta) \quad (12)$$

to ensure that the discussions are practical.

Now, we get

$$\Pi_{2,j} = \Pi_2 = \frac{n_1 [b(n_1 + 1)(n_2 + 1)^2 - n_1 g(n_2 + \beta - n_2\beta)^2] (a - v_1 - v_2^0)^2}{[b(n_1 + 1)(n_2 + 1)^2 - n_1 g(1 - \beta + n_2\beta)(n_2 + \beta - n_2\beta)]^2} \quad (13)$$

$$Q_1 = Q_2 = Q =$$

$$\frac{n_1 n_2 (n_2 + 1) (a - v_1 - v_2^0)}{[b(n_1 + 1)(n_2 + 1)^2 - n_1 g(1 - \beta + n_2\beta)(n_2 + \beta - n_2\beta)]}$$

$$q_{1,i} = q_1 =$$

$$\frac{n_2(n_2+1)(a-v_1-v_2^0)}{[b(n_1+1)(n_2+1)^2 - n_1g(1-\beta+n_2\beta)(n_2+\beta-n_2\beta)]}$$

$$q_{2,j} = q_2 =$$

$$\frac{n_1(n_2+1)(a-v_1-v_2^0)}{[b(n_1+1)(n_2+1)^2 - n_1g(1-\beta+n_2\beta)(n_2+\beta-n_2\beta)]}$$

$$\Pi_{1,i} = \Pi_1 = bq_1^2$$

Proposition 1 Each supplier's R&D expenditure increases with R&D efficiency and the number of the manufacturers and decreases with the number of the suppliers.

Proof From (11) we can immediately get $\frac{\partial x}{\partial g} > 0$ and $\frac{\partial x}{\partial n_1} > 0$. Thus, the R&D effort of the suppliers increases if R&D activities are more efficient or there are more manufacturers.

Partially differentiating (11) with respect to n_2 , we get

$$\text{Sign}\left\{\frac{\partial x}{\partial n_2}\right\} = \text{Sign}\{n_1g\beta(n_2+\beta-n_2\beta)^2 - b(n_1+1)(n_2+1)(n_2-n_2\beta+3\beta-1)\} \quad (14)$$

After some simple manipulations, we can show that

$$\begin{aligned} & b(n_1+1)(n_2+1)(n_2-n_2\beta+3\beta-1) \\ & \geq \frac{b(n_1+1)(n_2+1)^2(n_2+\beta-n_2\beta)\beta}{1-\beta+n_2\beta} \end{aligned} \quad (15)$$

always holds since it is equivalent to $(n_2-1)(2\beta-1)^2 \geq 0$, which is undoubtedly true.

Substituting the inequity in (12) into (15) yields

$$\begin{aligned} & b(n_1+1)(n_2+1)(n_2-n_2\beta+3\beta-1) \\ & > n_1g\beta(n_2+\beta-n_2\beta)^2 \end{aligned}$$

which leads to $\frac{\partial x}{\partial n_2} < 0$ from (14). Thus, each supplier

invests less in R&D if more firms exist in the upstream tier.

Proposition 2 (1) For large spillovers ($\beta \geq 0.5$), the suppliers reduce their R&D expenditures when the spillover increases. (2) We can not unambiguously indicate how the R&D investments vary with the spillover if $\beta < 0.5$. However, the less the numbers of the manufacturers and the suppliers, the lower the R&D efficiency and the higher the spillover are, the more likely that the R&D investments decrease.

Proof Partially differentiating (11) with respect to β , we have

$$\text{Sign}\left\{\frac{\partial x}{\partial \beta}\right\} = \text{Sign}\{n_1g(n_2+\beta-n_2\beta)^2 - b(n_1+1)(n_2+1)^2\} \quad (16)$$

For $\beta \geq 0.5$, we get

$$\begin{aligned} & b(n_1+1)(n_2+1)^2 > n_1g(1-\beta+n_2\beta)(n_2+\beta-n_2\beta) \\ & \geq n_1g(n_2+\beta-n_2\beta)^2 \end{aligned} \quad (17)$$

from (12). Thus, we obtain $\frac{\partial x}{\partial \beta} < 0$ from (16) and (17) for $\beta \geq 0.5$.

For $\beta < 0.5$, $\frac{\partial x}{\partial \beta} < 0$ holds only if

$$b(n_1+1)(n_2+1)^2 > n_1g(n_2+\beta-n_2\beta)^2 \quad (18)$$

which ensures that the condition in (12) satisfies. (18) can be rewritten as

$$\frac{b(n_1+1)}{n_1} > \frac{g(n_2+\beta-n_2\beta)^2}{(n_2+1)^2} \quad (19)$$

It can be seen that the left-hand side of (19) decreases with n_1 while the right-hand side decreases with β and increases with g and n_2 if $\beta < 0.5$. Hence, if $\beta < 0.5$, the greater β and the smaller n_1 , n_2 and g are, the more likely that x decreases with β .

Proposition 3 If an increase in the spillover occurs,

the quantity of each tier and each firm and the profit of each manufacturer increases if $\beta < 0.5$ and decreases if $\beta > 0.5$.

Proof It is easily to get

$$\begin{aligned}\text{Sign}\left\{\frac{\partial Q}{\partial \beta}\right\} &= \text{Sign}\left\{\frac{\partial q_1}{\partial \beta}\right\} = \text{Sign}\left\{\frac{\partial q_2}{\partial \beta}\right\} \\ &= \text{Sign}\left\{\frac{\partial \Pi_1}{\partial \beta}\right\} = \text{Sign}\{1 - 2\beta\}\end{aligned}$$

which leads to Proposition 3 immediately.

Proposition 4 (a) Each supplier's profit decreases with the spillover if $\beta \geq \frac{2n_2 - 1}{3(n_2 - 1)}$. (b) It can not generally concluded how the profit varies with the spillover if $\beta < \frac{2n_2 - 1}{3(n_2 - 1)}$. However, the greater g , n_1 , n_2 and $|0.5 - \beta|$ are, the more likely that the profit decreases with β .

Proof Partially differentiating (13) with respect to β leads to

$$\begin{aligned}\text{Sign}\left\{\frac{\partial \Pi_2}{\partial \beta}\right\} &= \text{Sign}\{b(n_1 + 1)(n_2 + 1)^2[(2n_2 - 1) - \\ &\quad 3(n_2 - 1)\beta] - n_1 g(n_2 + \beta - n_2 \beta)^3\}\end{aligned}\quad (20)$$

The term in the square brackets of the right-hand side of (20) is negative if $\beta \geq \frac{2n_2 - 1}{3(n_2 - 1)}$. This proves the first part of Proposition 4.

If $\beta < \frac{2n_2 - 1}{3(n_2 - 1)}$, we can get from (20) that

$$\begin{aligned}\text{Sign}\left\{\frac{\partial \Pi_2}{\partial \beta}\right\} &= \text{Sign}\{\varphi_1\} \\ \varphi_1 &\equiv \frac{b(n_2 + 1)^2[(2n_2 - 1) - 3(n_2 - 1)\beta]}{(n_2 + \beta - n_2 \beta)^3} - \frac{n_1}{n_1 + 1}g\end{aligned}$$

We have

$$\frac{\partial \varphi_1}{\partial g} < 0, \quad \frac{\partial \varphi_1}{\partial n_1} < 0, \quad \frac{\partial \varphi_1}{\partial n_2} < 0$$

$$\text{Sign}\left\{\frac{\partial \varphi_1}{\partial \beta}\right\} = \text{Sign}\{1 - 2\beta\}$$

Therefore, the greater g , n_1 , n_2 and $|0.5 - \beta|$ are, the more likely that the profit decreases with β .

Proposition 5 The quantity of each tier and each supplier increases with n_1 . Each incumbent supplier's quantity decreases if new suppliers enter the market, i.e., n_2 increases.

Proof It is easy to get

$$\frac{\partial Q}{\partial n_1} > 0, \quad \frac{\partial q_2}{\partial n_1} > 0$$

$$\begin{aligned}\text{Sign}\left\{\frac{\partial q_2}{\partial n_2}\right\} &= \text{Sign}\{-b(n_1 + 1)(n_2 + 1)^2 + \\ &\quad n_1 g[1 + (n_2 + 3)(n_2 - 1)(1 - \beta)\beta]\}\end{aligned}$$

Noting that

$$\begin{aligned}&n_1 g[1 + (n_2 + 3)(n_2 - 1)(1 - \beta)\beta] \\ &< n_1 g(1 - \beta + n_2 \beta)(n_2 + \beta - n_2 \beta)\end{aligned}$$

and the condition in (12), we get $\frac{\partial q_2}{\partial n_2} < 0$.

Proposition 6 (a) The total quantity of each tier increases with n_2 if $\beta = 0.5$. (b) It can not generally concluded how the number of the suppliers affects the quantity of each tier if $\beta \neq 0.5$. However, the greater n_1 , n_2 and $|0.5 - \beta|$ are, the more likely that the total quantity decreases with n_2 .

Proof It is easy to get

$$\text{Sign}\left\{\frac{\partial Q}{\partial n_2}\right\} = \text{Sign}\{\xi_1\},$$

$$\begin{aligned}\xi_1 &\equiv b(n_1 + 1)(n_2 + 1)^2 - \\ &\quad n_1 g[n_2^2 - (3n_2 + 1)(n_2 - 1)(1 - \beta)\beta]\end{aligned}$$

If $\beta = 0.5$, we can get $\xi_1 > 0$ from condition (12) and hence $\frac{\partial Q}{\partial n_2} > 0$.

If $\beta \neq 0.5$, noting that

$$n_1 g [n_2^2 - (3n_2 + 1)(n_2 - 1)(1 - \beta)\beta] \\ > n_1 g (1 - \beta + n_2 \beta)(n_2 + \beta - n_2 \beta)$$

we can not unambiguously sign ξ_1 from condition (12). However, if

$$\varphi_2 \equiv b - \frac{n_1 g}{n_1 + 1} \frac{n_2^2 - (3n_2 + 1)(n_2 - 1)(1 - \beta)\beta}{(n_2 + 1)^2} > 0$$

we can get $\frac{\partial Q}{\partial n_2} > 0$. Noting that $\frac{\partial \varphi_2}{\partial n_1} < 0$, $\frac{\partial \varphi_2}{\partial n_2} < 0$

and $\text{Sign}\{\frac{\partial \varphi_2}{\partial n_2}\} = \text{Sign}\{1 - 2\beta\}$, we know that the

greater n_1 , n_2 and $|0.5 - \beta|$ are, the more likely that Q decreases with n_2 .

Proposition 7 It can not generally concluded how the number of the manufacturers affects each incumbent manufacturer's quantity. However, the greater n_2 and $|0.5 - \beta|$ and the smaller g are, the more likely that each incumbent manufacturer's quantity decreases with n_1 .

Proof It is easy to get

$$\text{Sign}\{\frac{\partial q_1}{\partial n_1}\} = \text{Sign}\{\xi_2\}$$

$$\xi_2 \equiv g(1 - \beta + n_2 \beta)(n_2 + \beta - n_2 \beta) - b(n_2 + 1)^2 \quad (21)$$

We can not unambiguously indicate the sign of ξ_2 from (12) and (21). However, if

$$g(1 - \beta + n_2 \beta)(n_2 + \beta - n_2 \beta) - b(n_2 + 1)^2 < 0$$

i.e. $\varphi_3 \equiv \frac{g(1 - \beta + n_2 \beta)(n_2 + \beta - n_2 \beta)}{(n_2 + 1)^2} - b < 0$, we

have $\frac{\partial q_1}{\partial n_1} < 0$. Noting that $\frac{\partial \varphi_3}{\partial g} > 0$, $\frac{\partial \varphi_3}{\partial n_2} < 0$ and

$\text{Sign}\{\frac{\partial \varphi_3}{\partial n_2}\} = \text{Sign}\{1 - 2\beta\}$, we know that the greater

n_2 and $|0.5 - \beta|$ and the smaller g are, the more likely that q_1 decreases with n_1 .

4. R&D Cooperation

In this section, we consider the case that the suppliers cooperate in R&D and remain competition in production.

Each supplier j chooses its R&D expenditure x_j to

maximize $\Pi_{2,j} + \lambda \sum_{\substack{m=1 \\ m \neq j}}^{n_2} \Pi_{2,m}$, where $\lambda \in [0, 1]$

captures all possible degrees of R&D cooperation. In the extreme case of full cooperation (when $\lambda = 1$), each supplier chooses its R&D expenditure level to maximize their joint profits.

The decisions of the manufactures and suppliers on quantities keep the same as R&D competition in Section

2. However, supplier j chooses x_j to maximize

$\Pi_{2,j} + \lambda \sum_{\substack{m=1 \\ m \neq j}}^{n_2} \Pi_{2,m}$. From (10), we get

$$\frac{\partial \Pi_{2,k}}{\partial x_j} = \frac{n_1}{b(n_1 + 1)(n_2 + 1)^2} \left[(a - v_1 - v_2^0) + \right. \\ \left. (n_2 + \beta - n_2 \beta) \sqrt{g x_k} + (2\beta - 1) \sum_{\substack{m=1 \\ m \neq k}}^{n_2} \sqrt{g x_m} \right] \frac{(2\beta - 1)g}{\sqrt{g x_j}} \quad (22)$$

$$\frac{\partial \sum_{\substack{m=1 \\ m \neq j}}^{n_2} \Pi_{2,m}}{\partial x_j} = \frac{n_1}{b(n_1 + 1)(n_2 + 1)^2} \left[(n_2 - 1)(a - v_1 - v_2^0) + \right. \\ \left. (n_2 + \beta - n_2 \beta) \sum_{\substack{m=1 \\ m \neq j}}^{n_2} \sqrt{g x_m} + \right. \\ \left. (2\beta - 1)(\sqrt{g x_j} + (n_2 - 2) \sum_{m=1}^{n_2} \sqrt{g x_m}) \right] \frac{(2\beta - 1)g}{\sqrt{g x_j}} \quad (23)$$

Supplier j will choose its R&D investment level according to the following first-order condition

$$\frac{\partial (\Pi_{2,j} + \lambda \sum_{\substack{m=1 \\ m \neq j}}^{n_2} \Pi_{2,m})}{\partial x_j} = 0$$

We can get the first-order condition as the function of each supplier's R&D level from (22) and (23). Since the suppliers are identical, the subscript of x can be deleted and the first-order condition is reduced to

$$\frac{n_1 g[(n_2 + \beta - n_2 \beta) + \lambda(n_2 - 1)(2\beta - 1)]}{b(n_1 + 1)(n_2 + 1)^2 \sqrt{gx^*}} \times \left[(a - v_1 - v_2^0) + (1 - \beta + n_2 \beta) \sqrt{gx^*} \right] - 1 = 0$$

Then we get

$$\sqrt{gx^*} = \frac{n_1 g[(n_2 + \beta - n_2 \beta) + \lambda(n_2 - 1)(2\beta - 1)](a - v_1 - v_2^0)}{b(n_1 + 1)(n_2 + 1)^2 - n_1 g(1 - \beta + n_2 \beta)[(n_2 + \beta - n_2 \beta) + \lambda(n_2 - 1)(2\beta - 1)]} \quad (24)$$

We need the following condition

$$b(n_1 + 1)(n_2 + 1)^2 > n_1 g(1 - \beta + n_2 \beta)[(n_2 + \beta - n_2 \beta) + \lambda(n_2 - 1)(2\beta - 1)]$$

to ensure that the discussions are practical.

Substituting (24) into (7) yields

$$Q_1^* = Q_2^* = Q^* = \frac{n_1 n_2 (n_2 + 1)(a - v_1 - v_2^0)}{b(n_1 + 1)(n_2 + 1)^2 - n_1 g(1 - \beta + n_2 \beta)[(n_2 + \beta - n_2 \beta) + \lambda(n_2 - 1)(2\beta - 1)]} \quad (25)$$

We can also get

$$q_{1,i}^* = q_1^* = Q^* / n_1 \quad (26)$$

$$q_{2,j}^* = q_2^* = Q^* / n_2 \quad (27)$$

Proposition 8 The R&D expenditures increases with λ , the degree of R&D cooperation.

Proof The proof is immediately followed from the fact that $\frac{\partial \sqrt{gx^*}}{\partial \lambda} > 0$.

Noting that $\lambda = 0$ is actually the case that the suppliers do not cooperate in R&D, we know from Proposition 8 that the R&D investment level under cooperation is higher than that under competition. Furthermore, the larger the degree of cooperation is, the larger the gap of R&D level between cooperation and competition is. The gap reaches the largest value if the suppliers engage in full cooperation.

Proposition 9 (a) The R&D investments increase with the spillover if the degree of cooperation is appropriately large, i.e., $\lambda \geq 0.5$. (b) We can not generally indicate how the change in the spillover affects the R&D investment if $0 < \lambda < 0.5$. However, the greater λ and g and the smaller n_1 and β are, the more likely that the R&D investments increase with β . If $\beta < 0.5$, the greater n_2 is, the more likely that the R&D investment increases with β . If $\beta > 0.5$, the

smaller n_2 is, the more likely that the R&D investment increases with β .

Proof Differentiating (24) with respect to β yields

$$\text{Sign}\left\{\frac{\partial \sqrt{gx^*}}{\partial \beta}\right\} = \text{Sign}\{\xi_3\} \quad (28)$$

$$\xi_3 \equiv b(2\lambda - 1)(n_1 + 1)(n_2 + 1)^2 + n_1 g(1 - \beta + n_2 \beta)[(n_2 + \beta - n_2 \beta) + \lambda(n_2 - 1)(2\beta - 1)]^2 \quad (29)$$

(a) If $\lambda \geq 0.5$, it can be easily shown that $\xi_3 > 0$ and

hence $\frac{\partial \sqrt{gx^*}}{\partial \beta} > 0$.

(b) If $0 < \lambda < 0.5$, the first and second term on the right-hand side of (29) are, respectively, negative and positive. Thus, we can not unambiguously indicate the sign of ξ_3 . We can rewrite (28) as

$$\text{Sign}\left\{\frac{\partial \sqrt{gx^*}}{\partial \beta}\right\} = \text{Sign}\{\varphi_4\}$$

$$\varphi_4 \equiv g + \frac{b(n_1 + 1)}{n_1} \times \frac{(2\lambda - 1)(n_2 + 1)^2}{(1 - \beta + n_2 \beta)[(n_2 + \beta - n_2 \beta) + \lambda(n_2 - 1)(2\beta - 1)]^2}$$

We obtain

$$\frac{\partial \varphi_4}{\partial g} > 0, \quad \frac{\partial \varphi_4}{\partial n_1} < 0$$

$$\text{Sign}\left\{\frac{\partial \varphi_4}{\partial \beta}\right\} = \text{Sign}\{-(n_2 - 1)(2\lambda - 1)^2\}$$

$$\text{Sign}\left\{\frac{\partial \varphi_4}{\partial \lambda}\right\} = \text{Sign}\{(n_2 + \beta - n_2\beta) + (n_2 - 1)(2\beta - 1)(1 - \lambda)\}$$

$$\text{Sign}\left\{\frac{\partial \varphi_4}{\partial n_2}\right\} = \text{Sign}\{(1 - 2\beta)(2\lambda - 1)^2\}$$

Since $0 < \lambda < 0.5$, we can readily get $\frac{\partial \varphi_4}{\partial \beta} < 0$ and

$\frac{\partial \varphi_4}{\partial \lambda} > 0$. In addition, $\frac{\partial \varphi_4}{\partial n_2} > 0$ if $\beta < 0.5$ and

$\frac{\partial \varphi_4}{\partial n_2} < 0$ if $\beta > 0.5$.

Proposition 10 When the degree of R&D cooperation changes, the quantity of each tier and each firm increases if $\beta > 0.5$, decreases if $\beta < 0.5$ and remains unchanged if $\beta = 0.5$.

Proof From (25), (26) and (27), we have

$$\text{Sign}\left\{\frac{\partial Q}{\partial \lambda}\right\} = \text{Sign}\left\{\frac{\partial q_i}{\partial \lambda}\right\} = \text{Sign}\{2\beta - 1\}$$

which leads to the proposition immediately.

Proposition 11 (a) If $\lambda \geq \frac{n_2 - 1}{3n_2 - 1}$, the quantity of each tier and each firm increases with β . (b) If $\lambda < \frac{n_2 - 1}{3n_2 - 1}$, the quantity may increase or decrease with β . The smaller β is, the more likely that the quantity increases with β .

Proof From (25), (26) and (27), we get

$$\text{Sign}\left\{\frac{\partial Q}{\partial \beta}\right\} = \text{Sign}\left\{\frac{\partial q_i}{\partial \beta}\right\} = \text{Sign}\{\xi_4\}$$

$$\xi_4 \equiv (n_2 - 1) - \lambda(n_2 - 3) + 2(n_2 - 1)(2\lambda - 1)\beta$$

(a) If $\lambda \geq 0.5$, we have

$$\xi_4 \geq (n_2 - 1) - \lambda(n_2 - 3) > 0$$

If $\lambda < 0.5$, we can obtain

$$\begin{aligned} \xi_4 &\geq (n_2 - 1) - \lambda(n_2 - 3) + 2(n_2 - 1)(2\lambda - 1) \\ &= (3n_2 - 1)\lambda - (n_2 - 1) \end{aligned} \quad (30)$$

If $\frac{n_2 - 1}{3n_2 - 1} \leq \lambda < 0.5$, from (30) we get $\xi_4 \geq 0$ and the equation holds only when $\beta = 1$.

(b) If $\lambda < \frac{n_2 - 1}{3n_2 - 1}$, we obtain

$$\frac{\partial \xi_4}{\partial \beta} = 2(n_2 - 1)(2\lambda - 1) < 0$$

This completes the proof.

5. Conclusion

This paper has derived a model with upstream R&D in a two-tier supply chain. We considered R&D cooperation as well as R&D competition. Under R&D competition, we studied how the changes in R&D spillover, R&D efficiency and the numbers of the suppliers and the manufacturers affect R&D investments, quantities and profits. Under R&D cooperation, we showed that how R&D investments and quantities change with R&D spillover and the degree of R&D cooperation. The main results in the paper are given in eleven propositions.

Acknowledgement

This paper is supported by the National Natural Science Foundation of China (No. 70071016).

References

- [1] Veugelers, R., Vanden Houte, P., 1990. Domestic R&D in the presence of multinational enterprises. *International Journal of Industrial Organization* 8(1), 1-15
- [2] de Bondt, R., Slaets, P., Cassiman, B., 1992. The degree of spillovers and the number of rivals for maximum effective R&D. *International Journal of Industrial Organization* 10(1), 35-54
- [3] d'Aspremont, C., Jacquemin, A., 1988. Cooperative and noncooperative R&D in duopoly with spillovers. *American Economic Review* 78(5), 1133-1137

- [4] Suzumura, K., 1992. Cooperative and noncooperative R&D in an oligopoly with spillovers. *American Economic Review* 82(5), 1307-1320
- [5] Steurs, G., 1995. Interindustry R&D spillovers: What difference do they make? *International Journal of Industrial Organization* 13(2), 249-276
- [6] Anbarci, N., Lemke, R., Roy, S., 2002. Inter-firm complementarities in R&D: a re-examination of the relative performance of joint ventures. *International Journal of Industrial Organization* 20, 191-213
- [7] Qiu, L. D., Tao, Z., 1993. Policy on international R&D cooperation: Subsidy or tax? *European Economic Review* 42(9), 1727-1750
- [8] Petit, M. L., Sanna-Randaccio, F., 2000. Endogenous R&D and foreign direct investment in international oligopolies. *International Journal of Industrial Organization* 18(2), 339-367
- [9] Goel, R. K., 1999. R&D and output in a regulated vertically integrated oligopoly. *Bulletin of Economic Research* 51(4), 339-347
- [10] Corbett, C. J., Karmarkar U. S., 2001. Competition and structure in serial supply chains with deterministic demand. *Management Science* 47(7), 966-978
- [11] Abiru, M., Nahata, B., Raychaudhuri, S., Waterson, M., 1998. Equilibrium structures in vertical oligopoly. *Journal of Economic Behavior & Organization* 37, 463-480
- [12] Grandner, T., 2001. Unions in oligopolistic, vertically connected industries. *European Economic Review* 45, 1723-1740
- [13] Poyago-Theotoky, J., 1995. Equilibrium and optimal size of a research joint venture in an oligopoly with spillovers. *Journal of Industrial Economics* 43(2), 209-226
- [14] Žigić, K., Intellectual property rights violations and spillovers in North-South trade. *European Economic Review* 42(9), 1779-1799