Applying Fuzzy PERT in Cycle Time Management for Supply Chain System

Chen-Tung Chen Sue-Fen Huang

Department of Information Management, Da-Yeh University

112, Shan-Jiau Rd., Da-Tsuen, Changhua, Taiwan.

chtung@mail.dyu.edu.tw

ABSTRACT

As the fast technology evolution and the globalization trend, the competition among businesses transform into supply chain network gradually. However, the operation performance of the supply chain network influenced by many uncertain factors. These uncertain influence factors include the demand of the customer, the response time of delivery, operation lead time, stock level and cost. Under foregoing conditions, enterprises cannot control the variations in actual environment precisely and satisfy customers' requirements. Therefore, the linguistic variables are used to express these uncertainties in this paper. First, this paper presents a procedure to transform the operation processes of the supply chain network into a diagram to indicate the activities among members of supply chain. Second, combining fuzzy set theory with the program evaluation and review technique (PERT) to estimate the cycle time of supply chain system. According to the fuzzy PERT model, we can calculate the cycle time and find out the critical path of a supply chain system. Furthermore, we can compute the possibility of the order fulfillment of a supply chain system. Finally, a numerical simulation is presented to illustrate the procedure of this proposed model.

Keywords: Supply chain system, Fuzzy numbers, Program evaluation and review technique (PERT), Fuzzy PERT.

1. Introduction

Today the enterprise faces many rigorous and challenges, including the globalization of competition and cooperation, a varieties of customer requirements and shortened product life cycles. The enterprise has to look for the effective methods to adjust strategic and manageable models and maintain operations and competitive advantages. For example, Material Requirement Planning (MRP), Manufacturing Resource Planning (MRPII) and Enterprise Resource Planning (ERP) [10, 21] are used to integrate the operation processes and resources for enterprises. The purpose of these tools is to reduce the response time of the market demand and increase the customer satisfactions.

As the Internet and information technology grow rapidly, the industrial environment becomes more competitive for individual enterprise. Nowadays global marketplaces, no longer an individual enterprise competes as an independent entity but rather as an integral part of supply chain links [10]. In other words, each enterprise will depend on its management abilities to integrate and coordinate the complicated network of business relationships among supply chain members [5]. Therefore, supply chain management (SCM) has become one of the most important issues for enterprises.

A supply chain may be viewed as an integrated system that performs the procurement of raw material, its transformation to intermediate and end-products, distributor and promoting of the end-products to either retailers or customers [5, 21]. In recent years many researches are very interested in supply chain management problems and the concepts of SCM from different view points that were presented in [4, 5, 20, 27,28]. Unfortunately, there is no explicit description of supply chain management or its activities in the literature [30]. For example, New and Payne [23] described supply chain management as the chain linking each element of the manufacturing and supply processes from raw materials to the end user, encompassing several organizational boundaries. Jukka et al., [16] described supply chain management as a new way to provide goods and services to the end customer at the lowest cost and high service level, and the centric approach to manage the supply chain is no longer appropriate. SCM is an integrated approach to increase the effectiveness of the logistics chain by improving cooperation between members in the chain. Figure 1 shows the activities of a supply chain is comprised two fundamental procedures: (1) the Production layout and Inventory control process, and (2) the Distribution and Logistics process [1].

There is a growing interest in exploiting supply chain models in real applications and in developing decision support system to enhance supply chain management and control. Thus, a real supply chain operates in an uncertain environment. Different sources and types of uncertainty exist along the supply chain and make supply chain management problems more complex [6, 7, 20, 24, 25, 26]. Facing the dynamic states of the supply chain market, a supply chain network system that its internal and each operation of links in fast supply chain express effectively, and responds and makes more accurate forecasting to the market demand in time, is the problem that we want the important concerns the topic.

Program Evaluation and Review Technique (PERT) [3, 11, 12] is the management technique of a kind of project scheme and makes use of the mode of the operation network, marking whole scheme in the correlation of each operation activity. Making use of mathematics function compute possible complete times that are the earliest start time, the latest start time, the earliest finial time, the latest finial time, float time and total float time. Therefore, PERT is suitable to compute the order completion time and forecast the order fulfillment ability in the supply chain network system.

However, in traditional program evaluation and review technique, various dynamic activity durations must be the crisp number or obedient to certain probability distribution. Under the real situation, the operation time of each activity is usually difficult to define and estimate. Therefore, in recent years there are many researchers combing the concepts of the fuzzy set theory with program evaluation and reviews technique in developing the fuzzy program evaluation and review technique (FPERT)[3, 9, 12, 14, 15, 19, 22, 29]. In FPERT, the duration time of each activity is expressed by a fuzzy numbers.

In this paper, FPERT is applied to deal with the cycle time management problem of supply chain system. First, we can calculate the fuzzy completion time of the supply chain system by Fuzzy PERT method. Second, we will indicate the critical path of the supply chain system. Finally, the possibility of promise delivery can be calculated.

2. Fuzzy sets and notations

Zadeh introduced a theory whose objects fuzzy sets are sets with boundaries that are not precise. The membership in a fuzzy set is not a matter of affirmation or denial, but rather a matter of a degree[31].

A fuzzy set can be defined mathematically by assigning to each possible individual in the universe of discourse a value representing its grade of membership in the fuzzy set. This grade corresponds to the degree to which that individual is similar or compatible with the concept represented by the fuzzy set. Thus, individuals may belong in the fuzzy set to a greater or lesser degree as indicated by a larger or smaller membership grade. As already mentioned, these membership grades are very often represented by real number values ranging in the closed interval between 0 and 1[13].

2.1 Fuzzy Numbers

The fuzzy number \tilde{A} is a fuzzy set, its membership function $\mu_{\tilde{A}}(x)$, satisfies the following conditions [18]:

- 1. $\mu_{\tilde{A}}(x)$ is piecewise continuous.
- 2. $\mu_{\tilde{A}}(x)$ is convex fuzzy subset.
- 3. $\mu_{\tilde{A}}(x)$ is normality of a fuzzy subset, then the existence a real amount x_0 make $\mu_{\tilde{A}}(x_0) = 1$.

2.2 Triangular Fuzzy Number (TFN)

The triangle fuzzy number is a popular type of fuzzy number. A triangle fuzzy number can be expressed as $\tilde{T} = (l, m, u)$. When l > 0, then \tilde{T} is a positive triangle fuzzy number (PTFN)[8,32]. The membership function of positive triangle fuzzy number \tilde{T} is defined as (shown in Figure 2):

$$\mu_{\widetilde{T}}(x) = \begin{cases} \frac{x-l}{m-l} , l < x < m \\ \frac{u-x}{u-m} , m < x < u \\ 0 , otherwise \end{cases}$$
(1)

where l > 0.

2.3 The operation of the fuzzy numbers

Given two positive triangle fuzzy numbers $\tilde{T}_1 = (l_1, m_1, u_1)$ and $\tilde{T}_2 = (l_2, m_2, u_2)$, then the additive operation between them can be expressed as follows [17]:

$$\widetilde{T}_1 \oplus \widetilde{T}_2 = (l_1 + l_2, m_1 + m_2, u_1 + u_2)$$
 (2)

$$\tilde{T}_{1}\Theta\tilde{T}_{2} = (l_{1} - u_{2}, m_{1} - m_{2}, u_{1} - l_{2})$$
(3)

2.4 Ranking method of the fuzzy numbers

There are many ranking methods to transform fuzzy number into crisp value [2], Let $\tilde{T} = (l, m, u)$ be a triangle fuzzy number, then the defuzzied value is computed as [2]:

$$G(\tilde{T}) = \frac{l+m+u}{3} \tag{4}$$

Suppose \tilde{T}_1 and \tilde{T}_2 are two triangle fuzzy numbers, if $G(T_1) > G(T_2)$ then $\tilde{T}_1 > \tilde{T}_2$. If $G(T_1) = G(T_2)$ and $D(\tilde{T}_1) < D(\tilde{T}_2)$ then $\tilde{T}_1 > \tilde{T}_2$. If $G(T_1) = G(T_2)$ and $D(\tilde{T}_1) = D(\tilde{T}_2)$ then $\tilde{T}_1 \approx \tilde{T}_2$. Where $D(\tilde{T}_1) = u_1 - l_1$ and $D(\tilde{T}_2) = u_2 - l_2$.

2.5 The critical path method in fuzzy PERT

In this paper, we adopt an activity-on-node graph to represent a supply chain network. This graph includs two virtual tasks of null duration, start node and end node. For each task i the following notations ill be used:

- Succ(i)(respectively Pred(i))refers to the set of tasks that immediately follow task i , while SUCC(i)(resp. PRED(i)) refers to the set of all the tasks that come after task i;
- 2. d_i is the fuzzy duration of task i;
- 3. \tilde{t}_i is the fuzzy earliest starting dates of task i;
- 4. \tilde{T}_i is the fuzzy latest starting dates of task i;
- 5. \tilde{m}_i is the fuzzy float time of task i.

These variables are restricted in following relations:

$$\widetilde{t}_i = \max_{\substack{j \in \operatorname{Pr} ed(i)}} \widetilde{t}_j \oplus \widetilde{d}_i$$
(5)

$$\widetilde{T}_{i} = \min_{\substack{j \in Succ(i)}} \widetilde{T}_{j} \Theta \widetilde{d}_{i}$$
(6)

$$\widetilde{m}_i = \widetilde{T}_i \Theta \widetilde{t}_i \tag{7}$$

If $G(\tilde{m}_i) = 0$, then task i is a critical activity. The critical path is a path which connected these critical activities from start node and end node.

3. Order Fulfillment Analysis model

Suppose that there are 2 suppliers, 3 manufacturers, 3 distributors and a final retailer the supply chain system. The duration time from receiving the order to deliver materials to manufacturer of suppliers S_i (i=1, 2, ..., m) is denoted by $\tilde{T}_{S_i} = (S_{i1}, S_{i2}, S_{i3})$. The duration of the production of manufacturer M_k is denoted by $\tilde{T}_{M_i} = (M_{i1}, M_{i2}, M_{i3})$. The duration of the assembly

and distributor is denoted by $\tilde{T}_{D_i} = (D_{i1}, D_{i2}, D_{i3})$. The duration of the final retail is denoted by $\tilde{T}_{R_i} = (R_{i1}, R_{i2}, R_{i3})$.

Two possible cases in supply chain system are discussed as follows:

(1) The one supplier delivers materials to one manufacturer, one manufacturer products to one distributor, and the business sends the products to the final retail. In this case, the completion time of the supply chain system can be computed as:

$$\widetilde{t} = \widetilde{T}_{S_i} \oplus \widetilde{T}_{M_i} \oplus \widetilde{T}_{D_i} \oplus \widetilde{T}_{R_i}$$
(8)

(2) Several suppliers deliver materials to several manufacturer, several manufacturer delivers products to several distributor, and the business sends the products to the final retail. In this case, the completion time of the supply chain system can be computed as:

$$\widetilde{t} = \max\{\widetilde{T}_{S_i}, \widetilde{T}_{S_i}\} \oplus \max\{\widetilde{T}_{M_i}, \widetilde{T}_{M_j}\}$$

$$\oplus \max\{\widetilde{T}_{D_i}, \widetilde{T}_{D_j}\} \oplus \max\{\widetilde{T}_{R_i}, \widetilde{T}_{R_j}\}$$
(9)

Suppose the requirement due date of the customer is $RDD = \tilde{R} = (r_1, r_2, r_3)$, the completion time of supply chain system is denoted by $EF = \tilde{T}_{end} = (e_1, e_2, e_3)$, the promise delivery possibility (PDP) can be defined as(shown in Figure 3):

$$PDP = \begin{cases} 1 , & r_1 - e_3 \ge 0\\ \frac{\delta_1}{\delta_1 + \delta_2} , & r_1 - e_3 \le 0 \le r_3 - e_1 \\ 0 , & r_3 - e_1 \le 0 \end{cases}$$
(10)

where

$$\delta_1 = \int_{x \ge 0} \mu_{\widetilde{R} \Theta \widetilde{T}_{end}}(x) dx$$
 , $\delta_2 = \int_{x \le 0} \mu_{\widetilde{R} \Theta \widetilde{T}_{end}}(x) dx$ o

In other words, when the value δ_1 is larger, it represents the order fulfillment ability of supply chain system is higher. Thus, the responds ability of supply chain system is stronger.

Therefore, the order fulfillment ability analysis of supply chain system is described as follows:

- (1) If $r_1 e_3 \ge 0$, then the order fulfillment ability is 100%.
- (2) If $r_1 e_3 < 0 < r_3 e_1$, then the order fulfillment ability is denoted by PDP.
- (3) If $r_3 e_1 \le 0$, then the order fulfillment ability is

zero. In other words, the supply chain system can't delivery on time.

4. The results of simulation

In this paper, assumptions of supply chain processes considered are as follows:

- 1. The production facilities have unlimited capacities.
- 2. Customer demand is confined to a single product.
- 3. The raw material inventory is supplied from an external source.
- 4. External demand is fulfilled from the end-product inventory.

In order to implement the simulation analysis, we suppose the structure of the supply chain system as Figure 4. In Figure 4, S₁ and S₂ indicate the suppliers, M₁, M₂ and M₃ indicate the manufacturers, D₁, D₂ and D₃ indicates distributor, and the R₁ indicates the final retailer. Furthermore, the duration time of suppliers, manufacturers, distributor and the retailer are triangle fuzzy numbers and can be shown as Table 1. In Table 1, according to the LF- EF or LS- ES, we can compute the m value. According to the simple center method [2], we can compute the $G(\tilde{m}_i)$. If $G(\tilde{m}_i) = 0$, then task i is a critical activity. If completion time can't satisfy the customer requirement due date, we can adjust the duration time of activity on the critical path.

4.1 Requirement due date is crisp values

Suppose RDD=(20,20,20) and \tilde{T}_{end} =(13,18,25) then $\tilde{R}\Theta\tilde{T}_{end}$ = (-5,2,7). The PDP of supply chain network system can be computed as

$$PDP = \frac{\delta_1}{\delta_1 + \delta_2} = \frac{\frac{(25 - 13) \cdot 1}{2} - \frac{(25 - 20) \cdot 0.71}{2}}{\frac{(25 - 13) \cdot 1}{2}}$$
$$= 0.70$$

Suppose RDD=(15,15,15) and \tilde{T}_{end} =(13,18,25) then $\tilde{R}\Theta\tilde{T}_{end}$ = (-10,-3,2). The PDP of supply chain network system can be computed as

$$PDP = \frac{\delta_1}{\delta_1 + \delta_2} = \frac{\frac{(15 - 13) * 0.4}{2}}{\frac{(25 - 13) * 1}{2}} = 0.07$$

When $\tilde{T}_{end} = (13,18,25)$ and RDD are different crisp values, the PDP of a supply chain system can be computed as above definition in this paper. In this case, if \tilde{T}_{end} is unchanged and RDD is smaller then the PDP of supply chain system is lower.

4.2 Requirement due date is fuzzy numbers

(1) Suppose RDD=(12,20,23) and \tilde{T}_{end} =(13,18,25) then $\tilde{R}\Theta\tilde{T}_{end}$ = (-13,2,10). The PDP of supply chain network system can be computed as

$$PDP = \frac{\delta_1}{\delta_1 + \delta_2} = \frac{\frac{(13+10)*1}{2} - \frac{(0-(-13))*0.86}{2}}{\frac{(13+10)*1}{2}}$$
$$= 0.51$$

(2) Suppose RDD=(10,15,21) and \tilde{T}_{end} =(13,18,25) then $\tilde{R}\Theta\tilde{T}_{end}$ = (-15,-3,8). The PDP of supply chain network system can be computed as

$$PDP = \frac{\delta_1}{\delta_1 + \delta_2} = \frac{\frac{(0 - (-8)) * 0.73}{2}}{\frac{(8 + 15) * 1}{2}} = 0.25$$

(3) Suppose RDD=(15,18,21) and \tilde{T}_{end} =(13,18,25) then $\tilde{R}\Theta\tilde{T}_{end}$ = (-10,0,8). The PDP of supply chain network system can be computed as

$$PDP = \frac{\delta_1}{\delta_1 + \delta_2} = \frac{\frac{(0+8)*1}{2}}{\frac{(8+10)*1}{2}} = 0.44$$

When $\tilde{T}_{end} = (13,18,25)$ and RDD is a fuzzy number, the PDP of a supply chain system can be computed easily in this paper. According to the critical path, this method can indicate the critical activities and shorten their operation time to improve the PDP value. Therefore, the supply chain system can response the market requirement quickly and increase customers' satisfaction.

5. Conclusions

Along with the information technology growing fast, the competitions among enterprises have transformed into the competition between two supply chain systems. Thus, the robust management and operation model of supply chain system will increase the competitiveness for the enterprise. Besides, the order fulfillment ability of supply chain system is the key factor for increasing their competitive advantages.

In this paper, the triangle fuzzy numbers are used to stand for the various uncertainty of duration time. According to the FPERT, we can find out the fuzzy earliest / latest finial time and critical path in the supply chain network system quickly. In this paper the fuzzy model has proposed to analyze the order fulfillment ability of supply chain system for considering the uncertain duration time. According to the proposed model, we can immediately understand the status of each critical path of the supply chain system. If the completion time of critical path is not satisfy customer requirement due date, we can adjust the critical activities to raise PDP value and customer satisfaction. From the simulation results, it shows that if enterprise can hold duration time more precise, the order fulfillment ability of the supply chain network system more high.

This research is continued in two directions to further examine supply chain behavior in an uncertain environment.

(1) To include additional sources of uncertainty into the fuzzy model, such as rush time or rush cost.

(2) To examine the sensitivity of supply chain performances to various types of customer demand and fuzzy external supplier reliability.

(3) In the future, we can to creative new better critical path method application in fuzzy program evaluation and review technique in supply chain network system.

6. Acknowledgements

The authors grateful acknowledge the finance support of the National Science Council, Taiwan, under project numbers NSC 91-2745-P-212-001 and NSC 92-2213-E-212-008.

References

- Benita M. & Beamon, "Supply chain design and analysis: Models and methods," Int. J. Production Economics 55, 1998, 281-294.
- [2] Chen S. J. & Hwang C. L., Fuzzy Multiple Attribute Decision Making-Methods and Applications, Springer-Verlag Berlin Heidelberg, 1992.
- [3] Chen S.M. & Chang T.H., "Finding Multiple Possible Critical Paths Using Fuzzy PERT," IEEE transactions on systems, man, and cybernetics-part B:cybernetics, Vol. 31, No. 6, 2001, 930-937.
- [4] Christopher, M., Magrill, L. & Wills, G., "Educational development for marketing

logistics," International Journal of Physical Distribution and Logistics Management, 1998, 234-241.

- [5] Cooper M. C. & Lambert D. M., "Issues in Supply Chain Management," Industrial Marketing Management 29, 2000, 65-83.
- [6] Dejonckheere J., Disney S. M., Lambrecht M. R. & Towill D. R., "Transfer function analysis of forecasting induced bullwhip in supply chains," Int. J. Production Economics 76, 2002, 133-144.
- [7] Dolgui A. & Ould-Louly M. A., "A model for supply planning under lead time uncertainty," Int. J. Production Economics 78, 2002, 145-152.
- [8] Dubois D. & Prade H., Fuzzy Sets and Systems: Theory and Applications, Academy Press, 1980.
- [9] Dubois D., Fargier H. & Galvagonon V., "On latest starting times and floats in activity networks with ill-known durations," European Journal of Operational Research, 2003, 266-280.
- [10] Enns S. T., "MRP performance effects due to forecast bias and demand uncertainty," European Journal of Operational Research 138, 2002, 87-102.
- [11] Fargier H. & Galvagnon V., "Fuzzy PERT in series-parallel graphs," in 9th International Conference on Fuzzy Systems : Fuzzy IEEE, 2000, 717-722.
- [12] Fatemi Ghomi S.M.T. & Rabbani M., "A new structural mechanism for reducibility of stochastic PERT networks," European Journal of Operational Research, 2003, 394 – 402.
- [13] George J. Klir & Bo Yuan, "Fuzzy sets and Fuzzy logic," prentice hall international editions, 1995.
- [14] Hapke M. & Sloinski R., "Fuzzy project scheduling system for software development," Fuzzy Sets and Systems, 1994, 101-107.
- [15] Hapke M. and Sloinski R., "Fuzzy priority heuristics for project scheduling," Fuzzy Sets and Systems, 1996, 291-299.
- [16] Jukka K., Antti L. & Markku T., "An analytic approach to supply chain development," Int. J. Production Economics, 2001, 145-155.
- [17] Kaufmann A. & Gupta M. M., Introduction to fuzzy arithmetic: Theory and applications, International Thomson Computer Press, London, 1991.
- [18] Klir G. J. & Yuan B., Fuzzy Sets and Fuzzy Logic Theory and Applications, Prentice-Hall International Inc., 1995.
- [19] Kuchta D., "Use of fuzzy numbers in project risk (criticality) assessment," International Journal of Project Managemen, 2001t, 305-310.
- [20] Lee H. L., "Padamanabhan V. and Whang S., Information Distortion in a Supply Chain: The Bullwhip Effect," Management Science, Vol. 43, No. 4, 1997, 546-565.
- [21] Min H. & Zhou G., "Supply chain modeling: past,

present and future," Computers & Industrial Engineering 43, 2002, 231-249.

- [22] Mon D. L., Cheng C. H. and Lu H. C., "Application of fuzzy distributions on project management," Fuzzy Sets Systems 73, 1995, 227-234.
- [23] New S.J. & Payne P., "Research frameworks in logistics: three models, seven dinners and a survey," International Journal of Physical Distribution and Logistics Management 25 (10), 1995, 60-77.
- [24] Ouyang L. Y. & Chang H. C., "A minimax distribution free procedure for mixed inventory models involving variable lead time with fuzzy lost sales," Int. J. Production Economics 76, 2002, 1-12.
- [25] Petrovic D., "Simulation of supply chain behaviour and performance in an uncertain environment," Int. J. Production Economics 71, 2001, 429-438.
- [26] Petrovic D., Roy R. & Petrovic R., "Modelling and simulation of a supply chain in an uncertain environment," European Journal of Operational

Research 109, 1998, 299-309.

- [27] Ross D. F., Competing Through Supply Chain Management, Chapman and Hall, 1997.
- [28] Richard A. L., Michael F.S. & Hope J. S., "Strategic Internet application trends in supply chain management," Industrial Marketing Management 32,2003, 211-217.
- [29] S.M.T. Fatemi Ghomi & E. Teimouri, "Path critical index and activity critical index in PERT networks," European Journal of Operational Research, 2002, 147-152.
- [30] Tan K. C., "A framework of supply chain management literature," European Journal of Purchasing & Supply Management 7, 2001, 39-48.
- [31]Zadeh L. A., Fuzzy Sets, Information and Control, Vol. 8, 1965, 338-353.
- [32]Zimmerman H. J., Fuzzy Set theory and its applications, 2nd, Kluwer Academic Publishers Boston, 1991.



Figure 1. The supply chain process [1]



Figure 2. Positive triangle fuzzy number T.



Figure 3. Membership function of $\mu_{\tilde{R}\Theta\tilde{T}_{end}}$.



Figure 4. The graph of supply chain network.

node	Duration	Earliest	Earliest	Latest	Latest	Float time	
	time	start time	finial time	start time	finial time	(\widetilde{m})	$G(\widetilde{m})$
		(ES)	(EF)	(LS)	(LF)		
Supplier S ₁	(1,3,4)	(0,0,0)	(1,3,4)	(-5,7,19)	(-1,10,20)	(-5,7,19)	7
Supplier S ₂	(2,4,5)	(0,0,0)	(2,4,5)	(-12,0,12)	(-7,4,14)	(-12,0,12)	0^{*}
Manufacturer M ₁	(2,3,6)	(2,4,5)	(4,7,11)	(-1,10,20)	(5,13,22)	(-3,6,15)	6
Manufacturer M ₂	(3,4,7)	(2,4,5)	(5,8,12)	(-7,4,14)	(0,8,17)	(-9,0,9)	0^{*}
Manufacturer M ₃	(2,3,4)	(5,8,12)	(7,11,16)	(1,9,18)	(5,12,20)	(-4,1,6)	1
Distributor D ₁	(2,3,4)	(5,8,12)	(7,11,16)	(5,13,22)	(9,16,24)	(0,5,10)	5
Distributor D ₂	(3,4,5)	(5,8,12)	(8,12,17)	(0,8,17)	(5,12,20)	(-5,0,5)	0^*
Distributor D ₃	(4,4,4)	(8,12,17)	(12,16,21)	(5,12,20)	(9,16,24)	(-3,0,3)	0^{*}
Retailer R ₁	(1,2,4)	(12,16,21)	(13,18,25)	(9,16,24)	(13,18,25)	(-3,0,3)	0^{*}

Table 1 The stage duration time of supply chain system