

On the impact of electronic commerce on logistics operations

Harry Groenevelt
Simon School of Business
University of Rochester
Rochester, NY 14627
groenevelt@simon.rochester.edu

Pranab Majumder
Fuqua School of Business
Duke University
Durham, NC 27708
majumder@duke.edu

Abstract

This paper formulates and analyzes a model to integrate inventory management and promotion decisions in a multi-product environment. The model assumes that actual demands for the items depend on both item availability and the level of promotion used for the item. A notable feature of the model is that customer demand is partially backordered, where the fraction of demand backordered depends on how long a customer has to wait for delivery. The firm is assumed to have a limited promotion budget. The effect of a promotion is modeled through an increase in the demand rate of the item being promoted. We formulate a general, non-stationary, finite horizon version of the problem. However this problem is very difficult to solve optimally. In order to develop insights into the nature of the solution we formulate a stationary version of the general problem with the additional restriction that only one item can be promoted at a time. An efficient solution approach is developed for this stationary version, and limited numerical results are provided. These numerical results indicate that a coordinated approach to promotions and logistics decisions can lead to significantly higher profit for the firm.

1. Introduction

The emergence of internet based communication channels has enabled new levels of coordination in global manufacturing and logistics. Initiatives such as Collaborative Planning and Forecasting, electronic marketplaces, B2B and retailing over the web allow companies to develop new methods for designing and managing logistics systems. From the operations researcher's perspective, the challenge is in developing models that incorporate the additional data that is available in these environments and in using the data to implement more efficient and effective logistics policies. Since so much more data on customer behavior can be collected, we expect that these models will encompass more and more sophisticated modeling of customer behavior. This will allow a closer integration of marketing decisions with basic logistics decisions such as inventory management.

The goal of this paper is to develop a model to illustrate how marketing and operations decision-making can be integrated. The model we develop assumes that demand for items depends on both item availability and the level of promotion used for the item. We also assume that there is a limit on the amount of promotional activities

that the firm can engage in (this may reflect a limited promotion budget, a limit on the ability of consumers to absorb the promotional message, or a limit on the capacity – space on a web site – of the delivery vehicle for the promotional message). By choosing the promotions carefully and integrating the promotion schedule with inventory management the firm should be able to increase profits compared to a situation where inventory management and promotions are independently managed.

An important aspect of Internet retailing operations is the planning and coordination of advertising and promotions. Critical pages on a web site display featured items and certainly have an effect on the sales for these items. This behavior is similar to point-of-purchase displays in physical retailing, but is much more powerful since it can encompass any items that the retailer sells, not just those that satisfy certain size and selling price constraints.

Building upon the ability of e-commerce to capture more details of a customer's shopping behavior we present a scheme to coordinate customers purchase behavior with promotions and inventory stocking decisions. Our model shows how to coordinate the scarce resource of featured item advertising with inventory decisions in order to maximize the revenue from the assortment of items the retailer sells and to minimize the inventory costs.

Our model can be viewed as a generalization of the classical Economic Order Quantity model [10], and the related literature is vast, see e.g. [9] [23].

There is a small but growing literature that considers the impact of customer behavior on operations decisions. One strand of this body of work focuses on substitution behavior (see [1] [2] [7] [15] [21]).

Another strand focuses on promotion decisions. Cheng and Sethi [5] consider a periodic review model coupled with promotion decisions. Huchzermeier et al. [11] study the effect of consumer decisions when faced with promotions on the supply chain. Iyer and Jianming [12] show that information sharing is important in the context of retail promotions. In contrast to these papers we consider a continuous review model, explicitly consider stockouts as a cost reducing tactics, and consider multiple items in the context of limited promotions, as is the case for featuring items on web pages.

In our model we assume that actual demand depends on whether the item is in stock or not (in which case the demand rate further depends on the amount of time that will elapse until backorders can be filled). Some models where only a fraction of the demand is backordered when

a stockout occurs are examined in [16] [17] [20] [22].

Backorders have been the subject of extensive research in the context of stochastic models. An order point order quantity model with a mixture of backorders and lost sales is investigated in [19]. Lost sales will occur when backorders exceed a certain threshold level. General backorder costs in stochastic inventory models are discussed in [4], and in [3] backorder costs have fixed and proportional components.

Another approach to modeling the effect of stock-outs can be found in [18], where it is assumed that the demand rate is influenced by backorders. The approach of modeling backorders as dependent on the time until they can be filled was used by us in a somewhat different context in [8].

There is also extensive research on perishable inventories, when the stock at hand may decay, or become obsolete. One representative paper [14] integrates the stocking decision with backordering. While we do not have perishable inventories, we have “perishable” backorders, in the sense that backorders grow less than proportionately with the elapsed stockout time.

Finally, [13] uses an exponential pent-up demand function in the context of stochastic lead times. The focus is on algorithms to obtain the optimal stockout period without deriving any structural results. In this paper we model customer response to backorders through a general function, which allows various reaction curves to be fitted using available customer data. For details see [8].

The paper is structured as follows. In the next section we develop a general model for integrated promotion and inventory management over a finite time horizon. This model will be seen to be too complicated to solve optimally, hence we develop a simpler average cost version of the model in section 3. In section 4 we show how the single item average cost version of the model can be solved optimally, and in section 5 we will extend the analysis to the multiple item problem. Section 6 will discuss several numerical examples while in section 7 we give conclusions and ideas for further work in this area.

2. The Finite Horizon Model

We introduce the following notation.

N = the number of items in the assortment, indexed by i ,

\tilde{T} = length of the planning horizon,

D_i^H = demand rate for product i when it is being promoted,

D_i^L = demand rate for product i when it is not being promoted,

p_i = net margin (excluding inventory holding cost, fixed costs and promotions costs) per unit sold for item i ,

h_i = inventory holding cost (per unit per time period) for item i ,

F_i = fixed ordering cost for item i ,

$k_i(u)$ = fraction of demand that can be backordered

when item i is out of stock if the backorder will be filled in u time units, assumed to be non-increasing in u ,

c_i = cost per time unit of running the promotion for item i ,

w_i = promotional “weight” for item i , measures how much promotional capacity is consumed by fixed ordering cost for item i when it is being promoted,

W = total promotional capacity available (cannot be exceeded at any point in time),

$X_i(t) = 1$ if item i is being promoted at time t , 0 otherwise,

K_i = total number of replenishment orders placed during the planning horizon for item i ,

Q_i^k = order quantity for item i at the time that the k -th order is placed for item i ,

T_i^k = time that the k -th order is placed for item i ,

$S_i(t)$ = physical inventory for item i at time t ,

$\theta_i(t)$ = amount of time (measured from time t) until the next delivery is received for item i ,

$\hat{D}_i(t)$ = effective demand rate at time t for item i .

We assume that the demand for each item is deterministic. While there is physical inventory for item i , the demand rate is D_i^H while the item is being promoted, and D_i^L while it is not being promoted. If there is no physical inventory for item i , and it is u time units before the next delivery, the effective demand rate is $D_i^H k_i(u)$ while the item is being promoted and $D_i^L k_i(u)$ while it is not being promoted. Hence promotion affects the base demand rate, while physical inventory and the time until the next delivery affect the fraction of the base demand rate that is converted into sales. With these assumptions, the sales rate for item i at time t is equal to

$$\hat{D}_i(t) = \left(D_i^L + (D_i^H - D_i^L) X_i(t) \right) \times \left(I_{\{S_i(t) > 0\}} + k_i(\theta_i(t)) I_{\{S_i(t) \leq 0\}} \right). \quad (1)$$

The company will thus try to maximize

$$\sum_{i=1}^N \left(\int_{t=0}^{\tilde{T}} p_i \hat{D}_i(t) - c_i X_i(t) - h_i S_i(t) dt - K_i F_i \right), \quad (2)$$

subject to the following constraints.

Don't exceed the promotional capacity:

$$\sum_{i=1}^N w_i X_i(t) \leq W, \quad 0 \leq t \leq \tilde{T}. \quad (3)$$

Definition of sales:

$$\hat{D}_i(t) = \left(D_i^L + (D_i^H - D_i^L) X_i(t) \right) \times \left(I_{\{S_i(t) > 0\}} + k_i(\theta_i(t)) I_{\{S_i(t) \leq 0\}} \right), \quad (4)$$

$$0 \leq t \leq \tilde{T}; i = 1, \dots, N.$$

Physical inventory balance constraints:

$$S_i(u) = \left(S_i(0) + \sum_{k: T_i^k \leq u} Q_i^k - \int_{t=0}^u \hat{D}_i(t) dt \right)^+, \quad (5)$$

$$i = 1, \dots, N; 0 < u \leq \tilde{T}.$$

Order must clear the backlog:

$$\sum_{\xi=1}^k Q_i^{\xi} \geq \int_{t=0}^{T_i^k} \hat{D}_i(t) dt, \quad k=1, \dots, K_i; i=1, \dots, N. \quad (6)$$

Order sequencing:

$$T_i^k < T_i^{k+1}, \quad k=1, \dots, K_i-1; i=1, \dots, N. \quad (7)$$

Definition of θ_i :

$$\theta_i(t) = T_i^k - t, \quad T_i^{k-1} < t \leq T_i^k; k=1, \dots, K_i; i=1, \dots, N. \quad (8)$$

$X_i(t)$ is binary:

$$X_i(t) \in \{0,1\}, \quad 0 \leq t \leq \tilde{T}; i=1, \dots, N. \quad (9)$$

Clearly this problem is NP-Complete and would be extremely difficult to solve in the general case. Instead of attempting this (or developing heuristics for it) we instead formulate a simplified version in the next section.

3. A Simplified Model

In this section we will simplify the general problem formulated in the previous section in several ways. First, we will assume that only one item can be promoted at a time. This corresponds to assuming that $w_i = 1$ for all items and that $W = 1$. The second simplification is achieved by formulating an average profit version of the model, by assuming that all items are on the same replenishment cycle, and by requiring that an item can be promoted only once per cycle. Hence in this problem we have the following decision variables (in addition to the notation introduced earlier):

T = the common cycle length for all items,

ϕ_i = the amount of time that item i is promoted during the cycle, and

Q_i = the order quantity for item i .

Finally, we define

$\Pi_i(\phi_i, T)$ = maximum profit that can be achieved for item i in a cycle of length T when a single promotion of length ϕ_i is used and a single replenishment order is placed for item i .

We assume that a stationary policy is followed (clearly this is optimal in the simplified model), and since every item can be promoted only once per cycle, a feasible promotion schedule exists as long as total promotion time during a cycle does not exceed the length of the cycle. Hence the problem we want to solve can be formulated thus:

$$\Pi^* = \left\{ \begin{array}{l} \max \quad \frac{1}{T} \sum_{i=1}^N \Pi_i(\phi_i, T) \\ \text{s.t.} \quad \sum_{i=1}^N \phi_i \leq T \\ \phi_i \geq 0, \quad i=1, \dots, N \end{array} \right\}. \quad (10)$$

In the next section we will show how $\Pi_i(\phi_i, T)$ can be found efficiently for a given ϕ_i and T .

4. Solving the Single Item Sub-problem

In this section we will formulate the single item sub-problem, derive properties of the objective function, and give an efficient algorithm to solve it. Since we will

only consider one item here, we simplify the notation somewhat by dropping the subscript i in this section.

We will define a cycle to run from the moment the item runs out of stock (at time 0) until the item runs out of stock again (time T). Let x = the time at which the replenishment order arrives. During the cycle we can distinguish two distinct periods: when $0 \leq t < x$, backlog accumulates at a rate of $k(x-t)$ times the demand rate (which depends on whether a promotion is in effect). When $x \leq t < T$, sales occur directly from stock. (Note that in an extreme case it is possible that $x = 0$, i.e., the item is never stocked, the company takes orders and fills those orders when a replenishment order arrives.) The first issue to decide is when the promotion (which is assumed to have a length $\phi < T$) should be scheduled.

Proposition 1. *Let x (= the time during a cycle that the inventory runs out) be given. Let y = the start time of the promotion that maximizes the per cycle profit. Then*

$$y \leq x \leq y + \phi \leq T \quad (11)$$

Proof. It is tedious but not difficult to show that all the other cases are dominated by one that satisfies equation (11).

Next, define $z = x - y$. Then (11) is equivalent to

$$\max(0, \phi + x - T) \leq z \leq \phi \quad (12)$$

For given stockout time x and promotion start time y satisfying (11), the total units sold per cycle is given by

$$Q(x, z) = [T - K(x)] D^L + [\phi - K(z)] (D^H - D^L) \quad (13)$$

where

$$K(x) = x - \int_0^x k(t) dt, \quad (14)$$

and total inventory (in units of product \times time units) is

$$H(x, z) = \frac{1}{2} (T - x)^2 D^L + \frac{1}{2} (\phi - z)^2 (D^H - D^L). \quad (15)$$

Note that K is convex, and hence $Q(x, z)$ is jointly concave in x and z . Since $H(x, z)$ is jointly convex in x and z it follows that $\Pi(x, z) = pQ(x, z) - hH(x, z) - F$ is jointly concave.

Proposition 2. *The policy (\hat{x}, \hat{z}) that maximizes the profit per cycle for a given promotion length ϕ and cycle length T can be characterized as follows. If $hT \leq p(1 - k(0))$ then $\hat{x} = 0$, otherwise \hat{x} solves the equation*

$$k(x) - \frac{h}{p} x - \left(1 - \frac{hT}{p}\right) = 0. \quad (16)$$

If $h\phi \leq p(1 - k(0))$ then $\hat{z} = 0$, otherwise \hat{z} solves the equation

$$k(z) - \frac{h}{p} z - \left(1 - \frac{h\phi}{p}\right) = 0. \quad (17)$$

Proof: Note that

$$\frac{\partial \Pi(x, z)}{\partial x} = [h(T - x) - p(1 - k(x))] D^L, \quad (18)$$

$$\frac{\partial \Pi(x, z)}{\partial z} = [h(\phi - z) - p(1 - k(z))] (D^H - D^L). \quad (19)$$

Hence the first order conditions are equivalent to (16) and

(17). However, these don't give the solution if any of the constraints in (12) is violated. It is easy to show that the left hand sides of (16) and (17) are decreasing in x and z . If we substitute $z = \varphi$ in the LHS of (17) we obtain a non-positive number, and hence $\hat{z} \leq \varphi$. If we substitute $z = 0$ in the LHS of (17), we obtain a positive number as long as $h\varphi > p(1 - k(0))$, and hence in that case \hat{z} satisfies equation (17).

Substituting $x = T + \hat{z} - \varphi$ into equation (16) yields a non-positive number since the second term cannot exceed $k(\hat{z}) - 1$ by equation (17). Hence $x \leq T + \hat{z} - \varphi$. Finally, substituting in $x = 0$ into (16) concludes the proof.

To conclude this section, we prove the following proposition.

Proposition 3. Assume $\lim_{x \rightarrow \infty} \frac{1}{x} \int_0^x k(t)dt = 0$, then

- (i) $\lim_{T \rightarrow \infty} \frac{1}{T} \max \{\Pi(x, z)\} = 0$
- (ii) $\lim_{T \downarrow 0} \frac{1}{T} \max \{\Pi(x, z)\} = -\infty$.

Proof: To prove (i), first note that $Q(\hat{x}, \hat{z})/T$ is bounded from above by D^H as $T \rightarrow \infty$, while $H(\hat{x}, \hat{z})/T$ approaches ∞ as long as $(T - \hat{x})^2/T \rightarrow \infty$. So assume $(T - \hat{x})^2/T \leq U$ for all T . Then \hat{x} grows about as fast as T , and the assumption insures that $Q(\hat{x}, \hat{z})/T \rightarrow 0$ as $T \rightarrow \infty$. To prove (ii), note that $\Pi(\hat{x}, \hat{z}) \rightarrow -F$ as $T \rightarrow 0$.

5. Solving the Multi Item Problem

In this section we give an efficient algorithm to solve the multi-item problem (10) formulated in section 3. First of all, using the derivation in the previous section it is not hard to show that note that $\Pi_i(\varphi_i, T)$ is jointly concave in φ_i and T . Hence the objective function in (10) is quasi-concave. To solve (10) we adopt the following approach. Define

$$G(T) = \left\{ \begin{array}{l} \max \sum_{i=1}^N \Pi_i(\varphi_i, T) \\ \text{s.t.} \quad \sum_{i=1}^N \varphi_i \leq T \\ \varphi_i \geq 0 \ (i = 1, \dots, N) \end{array} \right\}, \quad (20)$$

$$f_i(u; S, D) = \left(p_i(S - K(u)) - \frac{1}{2} h_i(S - u)^2 \right) D, \quad (21)$$

$$g_i(S; \lambda, D) = \max \{ f_i(u; S, D) - \lambda S : u \geq 0 \}, \quad (22)$$

then

$$\Pi_i(\varphi, T) = g_i(T; 0, D_i^L) + g_i(\varphi; 0, D_i^H - D_i^L) - F_i, \quad (23)$$

and the Lagrangian of (20) can be written as follows.

$$G(T) = \sum_{i=1}^N g_i(T; 0, D_i^L) - \sum_{i=1}^N F_i + \min_{\lambda \geq 0} \left\{ \lambda T + \sum_{i=1}^N \max_{\varphi_i \geq 0} g_i(\varphi_i, \lambda, D_i^H - D_i^L) \right\} \quad (24)$$

Note that we can write the inner maximizations in (24) as follows:

$$\max_{\varphi_i \geq 0, z_i \geq 0} \left\{ \left(p_i(\varphi_i - K(z_i)) - \frac{1}{2} h_i(\varphi_i - z_i)^2 \right) (D_i^H - D_i^L) - \lambda \varphi_i \right\}, \quad (25)$$

and that the objective function in (25) is jointly concave. Using the first order conditions and (17), one readily shows that the optimal solution to (25) is given by

$$\left. \begin{array}{l} \hat{z}_i(\lambda) = 0, \hat{\varphi}_i(\lambda) = 0 \quad \text{if } \lambda r_i \geq 1 \\ \hat{z}_i(\lambda) = k_i^{-1}(\lambda r_i) \\ \hat{\varphi}_i(\lambda) = \hat{z}_i(\lambda) + \frac{p_i}{h_i}(1 - \lambda r_i) \end{array} \right\} \quad \text{otherwise} \quad (26)$$

where $r_i = 1/(p_i(D_i^H - D_i^L))$. Hence these sub-problems are particularly easy to solve. The second major term in (24) is therefore a standard Lagrangian optimization problem that is quite easy to solve numerically. Similarly, note that finding $g_i(T; 0, D_i^L)$ merely requires solving (16).

Therefore $G(T)$ can be readily found for any T . So we are finally ready to give the approach to solving the problem

$$\max \{G(T)/T : T > 0\}, \quad (27)$$

which of course is merely a different way of stating (10). Define $B(T; \alpha) = G(T) - \alpha T$. It is not hard to show the following

Proposition 4.

- (i) If $B(T; \alpha) > 0$ for some $\alpha \geq 0$ and $T > 0$ then $\Pi^* > \alpha$.
- (ii) If $\max \{B(T; \alpha) : T \geq 0\} \leq 0$ then $\Pi^* \leq \alpha$.
- (c) $B(T; \alpha)$ is concave in T .

Using this one can easily devise efficient algorithms to solve equation (10). The details will be left to the reader. For other examples of this approach see [6] [8].

6. Numerical results

In this section we will provide the results from a modest numerical study. In the study we assume that $k_i(u) = e^{-\mu_i u}$ throughout. Example 1 has $N = 4$ identical products with parameters as follows:

$$D_i^H = 20, D_i^L = 14, p_i = 3, h_i = 0.2, F_i = 10, \mu_i = 1 \quad (28)$$

For this case the optimal solution is

$$\begin{aligned} x_i &= 0.1862, z_i = 0.0436, \varphi_i = 0.6837, \\ T &= 2.735, \Pi_i = 39.17, \Pi = 156.69. \end{aligned} \quad (29)$$

First of all, it is interesting to see what happens when marketing and operations don't coordinate on setting the cycle length T . Figure 1 shows the total profit that is achieved when marketing arbitrarily chooses the cycle length and the inventory policy used is optimal given the cycle length.

The figure clearly shows a substantial penalty in terms of lost profit when the promotion cycle is too short, and a much less severe penalty when the promotion cycle is longer than optimal. Part of the steep decline in profits with a short cycle may be due to the model assumption that inventory cycles and promotion cycles must coincide.

When promotion cycles become very short, one replenishment order might be placed to cover two or more promotion cycles. We don't consider this possibility here.

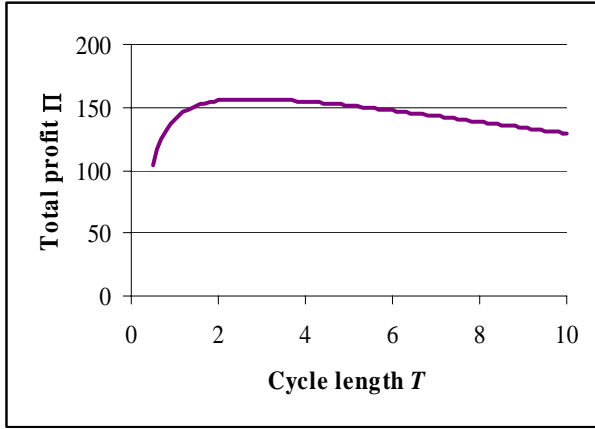


Figure 1: Maximum total profit given the cycle length T for Example 1.

Next, we consider the impact of the customers' willingness to wait for an out of stock item. This is captured in the model through the parameter μ of the function $k(u)$. The larger μ , the less willing customers are to wait. In the base case when $\mu = 1$, the fraction of customers that is willing to wait one time unit is equal to $\exp(-1) = 0.368$. In Figure 2 we show base case profit as a function of T for $\mu = 0.1$, $\mu = 0.5$ and $\mu = 2.0$, with other parameters as in (28).

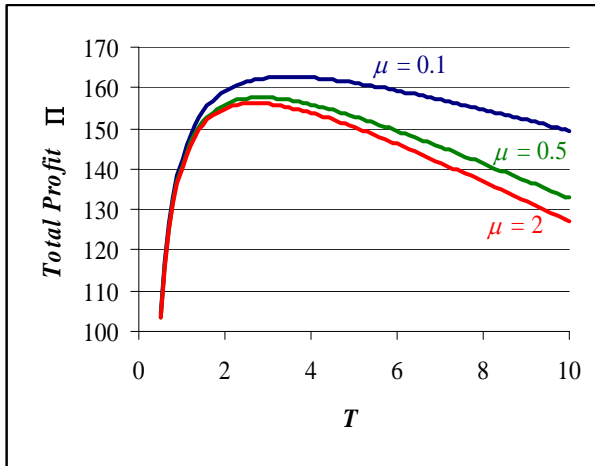


Figure 2. Impact of promotion cycle length T for varying customer willingness to wait for Example 1.

This shows that the profit is less sensitive to long promotion period lengths when customers are less sensitive to stockouts. For short promotion periods there is virtually no difference, which is accounted for by the fact that stockout durations are very short when the order cycle is short and hence fewer lost sales occur.

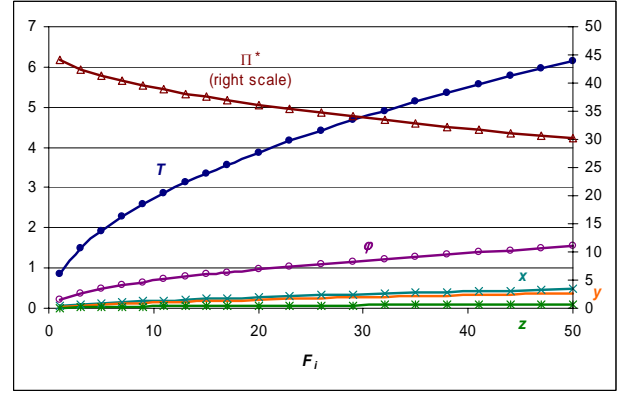


Figure 3: Optimal policy and profit as functions of fixed cost F_i for Example 1.

Next, we consider the impact of the fixed cost on the optimal policy and its cost, see Figure 3. This figure uses the base case parameters but varies the fixed cost F_i from 1 to 50. As expected, the optimal cycle length T increases as the fixed cost increases, and the optimal profit Π^* decreases. Since there are 4 products with identical parameters in this example, the optimal policies are identical as well. The promotion length ϕ is always equal to 25% of the cycle length T and the parameters x_i , y_i and z_i show little change when expressed as a proportion of the cycle length.

Finally, we will consider an asymmetrical example. The parameters for Example 2 are:

$$\begin{aligned} D_1^H &= 22, D_2^H = D_3^H = D_4^H = 20, \\ D_1^L &= 14, p_i = 3, h_i = 2, \mu_i = 1 \end{aligned} \quad (30)$$

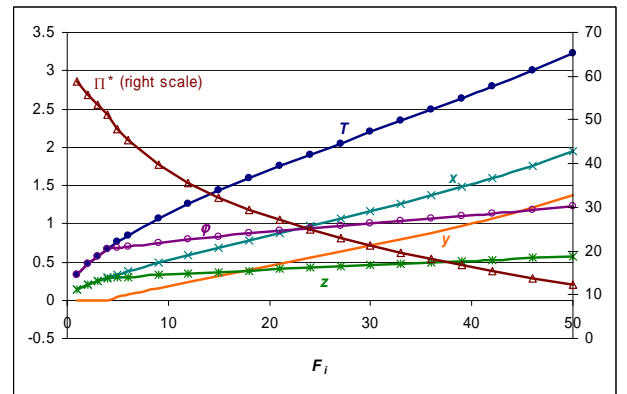


Figure 4. Optimal policy and profit for item 1 as functions of F_i for Example 2.

The optimal policies for item 1 are shown in Figure 4. In this example we see that when F_i is small (less than about 4), $y_1 = 0$, $\phi = T$ and $x_1 = z_1$, i.e., item 1 is always being promoted, even though the item is not always in stock. The duration of the stockout is short enough however to prevent serious loss sales. As the fixed cost F_i increases, increasing inventory holding costs make it more advantageous to incur more stockouts for item 1 and therefore it becomes less attractive to do promotion on item 1, which allows promotions to be scheduled for the other items as well. Note that the optimal profit for item 1 is not a convex function of F_i at the point where

promotions for the other items become attractive. The optimal policies and profits for items 2, 3 and 4 are depicted in Figure 5.

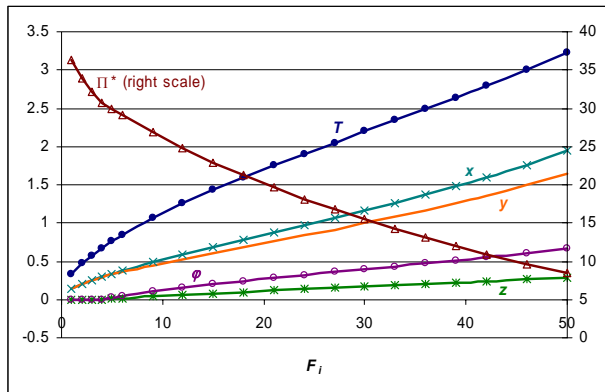


Figure 5. Optimal policy and profit as functions of F_i for items 2, 3 and 4 in Example 2.

Items 2, 3 and 4 are not scheduled for promotion until the fixed cost F_i exceeds about 4, and these items consume more and more of the total promotion budget as the fixed cost increases (at $F_i = 50$, about 40% of a cycle is used to promote item 1, and about 20% is used for each of the other three items). Note that the optimal policies in this example are neither convex nor concave functions of F_i .

7. Discussion

It is not hard to come up with interesting generalizations of our model. In the first place, the reaction of customers to promotions is very simple in our model. An obvious modification would be to make the sales rate during promotions a function of how long the promotion has been going on, similar to the backlogging rate being dependent on how long the customer has to wait until the item will be delivered. A second modification could be to allow several promotions to go on simultaneously, possibly accompanied by a segmentation of customer market. Both these modifications would make the model more realistic, but estimating the necessary parameters would become more difficult. We have demonstrated that it is quite possible to combine logistics aspects and marketing aspects in a comprehensive model. The advantage of this is obvious: as companies acquire better and more comprehensive information systems, much more details about customer preferences and behavior become available, and companies can use this information to improve both the service to their customers and their own bottom line.

The models in this paper are just the beginning, however. An operational model takes into account the specific information available in a practical situation, and uses this to make recommendations that tally with the business model that the company is trying to implement. This requires a lot of customization in the modeling approach. On the other hand, information system implementers need to take into account what information is needed to estimate and implement the more and more

sophisticated models that researchers develop. It is our hope that this paper can make a contribution to this necessary dialogue.

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