A Group Decision Making Approach for Dealing with Fuzziness in Decision Process

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Abstract

In order to deal with various imprecise opinions and preferences of decision makers in group decision-making process, this paper proposes a fuzzy group decision-making approach. The approach has three advantages from existing approaches. First, it can handle simultaneously group members' fuzzy preferences for alternative solutions, fuzzy judgments for solution selection criteria and fuzzy weights for their roles in group decision-making to arrive a group consensus decision. Second, it allows group members to generate selection criteria for the best solution rather than assume them to be given before a group meeting. The third is that it uses general fuzzy number to express linguistic terms which is used to describe the fuzziness of individual preferences, judgments and weights in group decision-making. It therefore accepts any forms of fuzzy number, including triangular fuzzy number, rectangle fuzzy number and continuous fuzzy number, when applying the group decision-making approach.

1. Introduction

As the complexity of organizational decision making increases the need for decision meetings and for working in groups increases [12]. A 'decision group' is the term for a "small, self-regulating, self-contained task-oriented work group" that "typically focuses on organizationally assigned decision-making tasks" [1]. Generally, the goal of a decision group is to determine an optimal solution to a decision problem called a group decision.

A group decision is often based on a set of optimal alternative solutions for a decision problem. The set of solutions can be generated by a suitable model and made by multiple decision makers [3]. A set of selection criteria is used for assessing, or ranking, the alternatives. These criteria can be generated by the group during the meeting, or determined before the meeting. alternatives. Group members can be each awarded a weighting. The final group decision will be made through aggregating group members' preferences on alternative solutions under their weights and judgments on selection criteria. The final decision is expected to be the most acceptable by the group.

In real environments, group decision-making has to confront various conditions [7]. As the process of decision-making is centered on human beings, with their inherent subjectivity and imprecision in the articulation of opinions, one of the conditions is fuzziness. There is much more in the fuzziness of group decision making process than just the fuzziness of models. The decision makers may be each awarded a fuzzy weighting, say, 'an important person' or 'a very important person'. The decision makers would like to express their preferences to alternatives by linguistic terms, such as 'good' or 'very good' [10]. The evaluation for alternatives is under a set of criteria and decision maker may have uncertain comparison results for these criteria individually. Particularly the decision-making procedure has to be performed through many uncertain negotiations among a group of decision makers [6]. Conflicts of interest are inevitable, and support for achieving consensus and compromise is required. Further information can trigger an alternative solution which appears preferable to the current best, or a new selection criterion is proposed.

Literature has shown that fuzzy mathematical models, in particular, fuzzy numbers are identified as the most proper way to deal with linguistic terms. A relatively practical introduction of fuzzy set theory [14] into conventional decision-making models was presented by Bellman et al. [2], Yager et al. [13] and Zimmermann [16]. Following this, further research was carried out in the area of fuzzy decision-making in the last twenty years. Existing fuzzy group decision-making approaches have handled respectively each of the three main aspects of the fuzziness in arriving a group decision-making: individual fuzzy preferences on alternative solutions (such as [5], [11]), individual judgments on solution selection criteria (such as [8], [4]), and individual roles in attempting to reach optimal solutions (such as [9]). Linguistic terms have been used in each of the three aspects of fuzziness in group decision-making [10][15].

However, there are two limitations in existed research. The first limitation is that the fuzziness regarding personal weights, alternatives and criteria in group decision-making

is studied respectively, but they can exist simultaneously. Another is that the fuzziness in generating criteria during a meeting has less attention. The third limitation is that only triangular number form of fuzzy number is used to describe linguistic terms in related algorithms. Using triangular numbers to express fuzzy values is a simple way and easy to be calculated, but it cannot describe wholly all characteristics of fuzzy values. Particularly, many fuzzy values cannot be expressed by triangular numbers.

This study proposes a fuzzy group decision-making approach to handle wholly the three fuzzy properties discussed above, support selection criteria generation during a group meeting, and allow various forms of fuzzy numbers to be used. The approach is designed to achieve the following goals: (1) to make the final solution to reflect the various roles of group members in a decision-making process; (2) to reach a 'better' solution by aggregating the various opinions of group members; (3) to improve the decision quality by allowing the knowledge expression by linguistic terms; (4) to suit more real situations of group decision-making by support criteria generation, and (5) to generalize the applications of fuzzy group decision-making approaches by allowing any forms of fuzzy numbers to describe the fuzziness.

2. An integrated multi-criteria fuzzy group decision-making approach

Let $\boldsymbol{S} = \{\boldsymbol{S}_1, \boldsymbol{S}_2, \ldots, \boldsymbol{S}_m\}$, $m \geq 2$, be a given finite set of solutions for a decision problem and $\boldsymbol{P} = \{\boldsymbol{P}_1, \boldsymbol{P}_2, \ldots, \boldsymbol{P}_n\}$, $n \geq 2$, be a given finite set of decision makers. The proposed approach consists of seven steps within two levels:

Level 1: *selection criteria generation*

Step 1: Each decision maker $\boldsymbol{P}_k$ (k = 1, 2, ..., n) proposes one or more selection criteria $(C_{k_1}^k, C_{k_2}^k, \cdots, C_{k_p}^k)$. These criteria are then put into a criterion pool. These criteria are sorted based on the number of group members proposing the criterion for each criterion. Finally top-T criteria, $\boldsymbol{C} = \{\boldsymbol{C}_1, \boldsymbol{C}_2, \ldots, \boldsymbol{C}_t\}$, are chosen to be selection criteria for the group.

Level 2: *Individual Preference Generation*

Step 2: Decision maker $\boldsymbol{P}_k$ (k = 1, 2, ..., n) determines the weight of selection criterion $\boldsymbol{C}$ by using the Analytic Hierarchy Process (AHP) method.

By pairwise comparison of the relative importance of selection criteria, the pairwise comparison matrix $\boldsymbol{E} = [\tilde{e}_{ij}^k]_{t\times t}$ is established, where $\tilde{e}_{ij}^k$ represents quantified judgments on pairs of selection criteria $\boldsymbol{C}_i$ and $\boldsymbol{C}_j$ or "*". The comparison scale belongs to a set of linguistic terms that contain various degrees of preferences required b y the decision maker $\boldsymbol{P}_k$ (k = 1, 2, ..., n). The linguistic terms for variable "preference" are show in Table 1 or "*" represents decision maker $\boldsymbol{P}_k$ (k = 1, 2, ..., n) don't know the relative importance of selection criteria $\boldsymbol{C}_i$ and $\boldsymbol{C}_j$.

Table 1 Linguistic terms for the comparison scale of selection criteria

Linguistic terms	Fuzzy numbers
Absolutely not important (ANI)	a_1
Not important (SNI)	a_2
Less important (WNI)	a_3
Important (EI)	a_4
More important (WI)	a_5
Much more important (SI)	a_6
Absolutely important (AI)	a_7

Consistent weights w_i^k $(i = 1, 2, \cdots, t)$ for every selection rule can be determined by calculating the geometric mean of that e_{ij}^k $(j = 1, 2, \cdots, i_k)$ is no "*" of each row of the matrix $[\tilde{e}_{ij}^k]_{t\times t}$, and then the resulting fuzzy numbers are normalized and denoted as $\tilde{w}_1^k, \tilde{w}_2^k, \cdots, \tilde{w}_t^k$, where $\tilde{w}_i^k \in F_T^*(R)$ and

$$\tilde{w}_i^k = \frac{w_i^k}{\sum_{i=1}^{t} w_i^k}, \quad \text{for } i = 1, 2, \cdots, t;\ k = 1, 2, \cdots, n.$$

Step 3: Against every selection rule $\boldsymbol{C}_j$ (j = 1, 2, ..., t), a belief level can be introduced to express the possibility of selecting a solution (i) under rule (j) for a decision maker (k). The belief level b_{ij}^k $(i = 1, 2, \cdots, t,\ j = 1, 2, \cdots, m, k = 1, 2, \ldots, n)$ belongs to a set of linguistic terms that contain various degrees of preferences required by a decision maker $\boldsymbol{P}_k$ (k = 1, 2, ..., n) or "**". The linguistic terms for variable "judgment" are shown in Table 2 or "**" represents the decision maker $\boldsymbol{P}_k$ (k = 1, 2, ..., n) don't know a belief level for expressing the possibility of selecting a solution (i) under rule (j).

Table 2 Linguistic terms for the belief levels of selection criteria

Linguistic terms	Fuzzy numbers
Very low (VL)	b_1
Low (L)	b_2
Medium low (ML)	b_3
Medium (M)	b_4
Medium high (MH)	B_5
High (H)	b_6
Very high (VH)	b_7

Step 4: Belief level matrix $\left(b_{ij}^k\right)(k = 1, 2, \cdots, n)$ is aggregated in to belief vector $\left(\overline{b}_j^k\right)$ $(j = 1, 2, \ldots, m, k = 1, 2, \ldots, n)$.

$$\overline{b}_j^k = \tilde{w}_{j_1}^k * b_{jj_1}^k + \tilde{w}_{j_2}^k * b_{jj_2}^k + \cdots + \tilde{w}_{j_s}^k * b_{jj_s}^k,$$

where $b_{jj_i}^k$ $(i = 1, 2, \cdots, s)$ is no "**".

Based on belief vectors, the decision maker $\boldsymbol{P}_k$ (k = 1, 2, ..., n) can make an overall judgment on the solutions,

called an individual selection vector. All individual selection vectors can compose a group of selection matrixes $\left(\bar{b}_j^k\right)_{n\times m}$.

Level 3: *Group Aggregation*

Step 5: As group members play different roles in an organization the relative importance of each decision maker may not equal in a decision group. Some are more important than the others. Therefore, the relative importance weighting of each decision maker should be considered. The linguistic terms $\tilde{v}_k$, $k = 1, 2, \cdots, n$ are shown in Table 3.

Table 3 Linguistic terms for the weights of decision makers

Linguistic terms	Fuzzy numbers
General decision person (GP)	c_1
Weakly important person (WP)	c_2
Strongly important person (SP)	c_3
The most important person (TP)	c_4

The normalized weight of a decision maker $\boldsymbol{P}_k$ (k = 1, 2, ..., n) is denoted as

$$\tilde{v}_k^* = \frac{\tilde{v}_k}{\sum_{i=1}^n v_{i0}^R}, \quad \text{for } k = 1, 2, \cdots, n.$$

Step 6: Considering the weights of all decision makers in a group, we can construct the weighted normalized fuzzy decision vector

$$(\tilde{r}_1, \tilde{r}_2, \cdots, \tilde{r}_m) = (\tilde{v}_1^*, \tilde{v}_2^*, \cdots, \tilde{v}_n^*)\left(\bar{b}_i^j\right)_{m\times n},$$

where $\tilde{r}_j = \sum_{k=1}^n \tilde{v}_k^* \bar{b}_j^k$.

Step 7: In the weighted normalized fuzzy decision vector the elements $\tilde{v}_j$, $j = 1,2,\cdots,m$ are normalized positive fuzzy numbers, and their ranges belong to closed interval [0, 1]. We can then define fuzzy positive-ideal solution (FPIS, r^*) and fuzzy negative-ideal solution (FNIS, r^-) as:

$$r^* = 1 \quad \text{and} \quad r^- = 0.$$

The positive and negative solution distances between each $\tilde{r}_j$ and r^*, $\tilde{r}_j$ and r^- can be calculated as: $d_j^* = d(\tilde{r}_j, r^*)$ and $d_j^- = d(\tilde{r}_j, r^-)$, $j = 1, 2, \cdots, m$

where $d(\tilde{a}, \tilde{b}) = \left(\int_0^1 \frac{1}{2}[(a_l^L - b_l^L)^2 + (a_l^R - b_l^R)^2]dl\right)^{\frac{1}{2}}$ is the distance measurement between two fuzzy numbers.

Step 8: A closeness coefficient is defined to determine the ranking order of all solutions once the d_j^* and d_j^- of each decision solution $\boldsymbol{S}_j$ (j = 1, 2, ..., m) are obtained. The closeness coefficient of each solution is calculated as:

$$CC_j = \frac{1}{2}\left(d_j^* + (1 - d_j^-)\right), \quad j = 1, 2, \cdots, m.$$

The solution $\boldsymbol{S}_j$ that corresponds to the largest CC_j is the best satisfactory solution of the decision group. If the solution cannot be accepted by the group, two actions can be taken. One is to alter the selection criteria. For example, add or remove a selection criterion. Another is to remove one or more of the worst alternative solutions and redo the process. The 'worst' solution is that which corresponds to the $Min\{CC_j ; j = 1, 2, \ldots, m\}$.

3. An Application of the Approach

Two solutions S_1, S_2 have been obtained as alternatives by a group which is organized by three members, P_1, P_2 and P_3. The group will select one from S_1, S_2 under three selection criteria C_1, C_2, C_3. The three members have different weights and are allowed to express their individual fuzzy preferences for the two solutions and imprecise judgments for the goals o f the three criteria using linguistic terms. The approach allows any forms of fuzzy numbers to describe these linguistic variables. Table 4, 5 and 6 show what we used.

Table 4 Linguistic variables and related fuzzy numbers for the comparison scale of selection criteria

Linguistic variables	Fuzzy numbers
Absolutely not important(ANI)	$\bigcup_{l\in[0,1]} l[0, \frac{\sqrt{1-l}}{10}]$
Not important (SNI)	$\bigcup_{l\in[0,1]} l[\frac{\sqrt{l}}{10}, \frac{\sqrt{9-8l}}{10}]$
Less important (WNI)	$\bigcup_{l\in[0,1]} l[\frac{\sqrt{8l+1}}{10}, \frac{\sqrt{25-16l}}{10}]$
Important (EI)	$\bigcup_{l\in[0,1]} l[\frac{\sqrt{16l+9}}{10}, \frac{\sqrt{49-24l}}{10}]$
More important (WI)	$\bigcup_{l\in[0,1]} l[\frac{\sqrt{24l+25}}{10}, \frac{\sqrt{81-32l}}{10}]$
Much more important (SI)	$\bigcup_{l\in[0,1]} l[\frac{\sqrt{32l+49}}{10}, \frac{\sqrt{100-19l}}{10}]$
Absolutely important (AI)	$\bigcup_{l\in[0,1]} l[\frac{\sqrt{19l+81}}{10}, 1]$

Table 5 Linguistic variables and related fuzzy numbers for the belief levels of selection criteria

Linguistic variables	Fuzzy numbers
Very low (VL)	$\bigcup_{l\in[0,1]} l[0, \frac{\sqrt{1-l}}{10}]$
Low (L)	$\bigcup_{l\in[0,1]} l[\frac{\sqrt{l}}{10}, \frac{\sqrt{9-8l}}{10}]$
Medium low (ML)	$\bigcup_{l\in[0,1]} l[\frac{\sqrt{8l+1}}{10}, \frac{\sqrt{25-16l}}{10}]$

Medium (M)	$\bigcup_{l\in[0,1]} l[\frac{\sqrt{16l+9}}{10}, \frac{\sqrt{49-24l}}{10}]$
Medium high (MH)	$\bigcup_{l\in[0,1]} l[\frac{\sqrt{24l+25}}{10}, \frac{\sqrt{81-32l}}{10}]$
High (H)	$\bigcup_{l\in[0,1]} l[\frac{\sqrt{32l+49}}{10}, \frac{\sqrt{100-19l}}{10}]$
Very high (VH)	$\bigcup_{l\in[0,1]} l[\frac{\sqrt{19l+81}}{10}, 1]$

Table 6 Linguistic variables and related fuzzy numbers for the weights of decision persons

Linguistic variables	Fuzzy numbers
General decision person (GP)	$\bigcup_{l\in[0,1]} l[\frac{\sqrt{16l+9}}{10}, \frac{\sqrt{49-24l}}{10}]$
Weakly important person (WP)	$\bigcup_{l\in[0,1]} l[\frac{\sqrt{24l+25}}{10}, \frac{\sqrt{81-32l}}{10}]$
Strongly important person (SP)	$\bigcup_{l\in[0,1]} l[\frac{\sqrt{32l+49}}{10}, \frac{\sqrt{100-19l}}{10}]$
The most important person (TP)	$\bigcup_{l\in[0,1]} l[\frac{\sqrt{19l+81}}{10}, 1]$

Step 1: by using the AHP method, three pairwise comparison matrices are established:

$$E^1 = E^2 = E^3 = \begin{pmatrix} EI & EI & * \\ EI & EI & * \\ * & * & EI \end{pmatrix} = \begin{pmatrix} a_4 & a_4 & * \\ a_4 & a_4 & * \\ * & * & a_4 \end{pmatrix}$$

Through computing the geometric mean of each row of the matrices, the normalized resulting numbers are obtained. As

$$\begin{pmatrix} w_1^1 \\ w_2^1 \\ w_3^1 \end{pmatrix} = \begin{pmatrix} w_1^2 \\ w_2^2 \\ w_3^2 \end{pmatrix} = \begin{pmatrix} w_1^3 \\ w_2^3 \\ w_3^3 \end{pmatrix} = \begin{pmatrix} \sqrt{a_4 a_4} \\ \sqrt{a_4 a_4} \\ a_4 \end{pmatrix}$$

$$= \begin{pmatrix} a_4 \\ a_4 \\ a_4 \end{pmatrix} = \begin{pmatrix} \bigcup_{l\in[0,1]} l[\frac{\sqrt{16l+9}}{10}, \frac{\sqrt{49-24l}}{10}] \\ \bigcup_{l\in[0,1]} l[\frac{\sqrt{16l+9}}{10}, \frac{\sqrt{49-24l}}{10}] \\ \bigcup_{l\in[0,1]} l[\frac{\sqrt{16l+9}}{10}, \frac{\sqrt{49-24l}}{10}] \end{pmatrix},$$

and $\sum_{i=1}^{3} w_{i0}^{1R} = \sum_{i=1}^{3} w_{i0}^{2R} = \sum_{i=1}^{3} w_{i0}^{3R} = 2.1$. So we have

$$\begin{pmatrix} \tilde{w}_1^1 \\ \tilde{w}_2^1 \\ \tilde{w}_3^1 \end{pmatrix} = \begin{pmatrix} \tilde{w}_1^2 \\ \tilde{w}_2^2 \\ \tilde{w}_3^2 \end{pmatrix} = \begin{pmatrix} \tilde{w}_1^3 \\ \tilde{w}_2^3 \\ \tilde{w}_3^3 \end{pmatrix} = \frac{1}{2.1} \begin{pmatrix} a_4 \\ a_4 \\ a_4 \end{pmatrix}$$

Step 2: Assesses

$$\begin{pmatrix} b_{11}^1 & b_{12}^1 & b_{13}^1 \\ b_{21}^1 & b_{22}^1 & b_{23}^1 \end{pmatrix} = \begin{pmatrix} M & VL & ** \\ VH & ** & M \end{pmatrix} = \begin{pmatrix} b_4 & b_1 & ** \\ b_7 & ** & b_4 \end{pmatrix},$$

$$\begin{pmatrix} b_{11}^2 & b_{12}^2 & b_{13}^2 \\ b_{21}^2 & b_{22}^2 & b_{23}^2 \end{pmatrix} = \begin{pmatrix} ** & M & VL \\ M & VH & ** \end{pmatrix} = \begin{pmatrix} ** & b_4 & b_1 \\ b_4 & b_7 & ** \end{pmatrix},$$

$$\begin{pmatrix} b_{11}^3 & b_{12}^3 & b_{13}^3 \\ b_{21}^3 & b_{22}^3 & b_{23}^3 \end{pmatrix} = \begin{pmatrix} VL & ** & M \\ ** & M & VH \end{pmatrix} = \begin{pmatrix} b_4 & ** & b_1 \\ ** & b_4 & b_7 \end{pmatrix}$$

Step 3: We have

$$\bar{b}_1^1 = \tilde{w}_1^1 b_{11}^1 + \tilde{w}_2^1 b_{12}^1 = \frac{1}{2.1}(a_4{}^2 + a_4 a_1),$$

$$\bar{b}_2^1 = \tilde{w}_1^1 b_{21}^1 + \tilde{w}_3^1 b_{23}^1 = \frac{1}{2.1}(a_4{}^2 + a_4 a_7),$$

$$\bar{b}_1^2 = \tilde{w}_2^2 b_{12}^2 + \tilde{w}_3^2 b_{13}^2 = \frac{1}{2.1}(a_4{}^2 + a_4 a_1),$$

$$\bar{b}_2^2 = \tilde{w}_1^2 b_{21}^2 + \tilde{w}_2^2 b_{22}^2 = \frac{1}{2.1}(a_4{}^2 + a_4 a_7),$$

$$\bar{b}_1^3 = \tilde{w}_1^3 b_{11}^3 + \tilde{w}_3^3 b_{13}^3 = \frac{1}{2.1}(a_4{}^2 + a_4 a_1),$$

$$\bar{b}_2^3 = \tilde{w}_2^3 b_{22}^3 + \tilde{w}_3^3 b_{23}^3 = \frac{1}{2.1}(a_4{}^2 + a_4 a_7).$$

Step 4: Assesses: The important weight v_1 of $\boldsymbol{P}_1$ is *TP*; the important weight v_2 of $\boldsymbol{P}_2$ is *GP* and the important weight v_3 of $\boldsymbol{P}_3$ is *GP*. Then

$$v_1 = c_4,\ v_2 = v_3 = c_1, \text{ and } \sum_{i=1}^{3} v_{i0}^R = 2.4,$$

therefore

$$v_1^* = \frac{c_4}{2.4} = \frac{a_7}{2.4},\ v_2^* = v_3^* = \frac{c_1}{2.4} = \frac{a_4}{2.4}.$$

Step 5: We have

$$\tilde{r}_1 = v_1^* \bar{b}_1^1 + v_2^* \bar{b}_1^2 + v_3^* \bar{b}_1^3 = \frac{1}{2.1\times 2.4}(a_7 + 2a_4)(a_4{}^2 + a_4 a_1)$$

$$= \bigcup_{l\in[0,1]} l[\frac{(16l+9)\sqrt{19l+81} + 2(16l+9)^{\frac{3}{2}}}{5040},$$

$$\frac{10(49-24l) + 10\sqrt{(1-l)(49-24l)} + 2(49-24l)^{\frac{3}{2}}}{5040}$$

$$+ \frac{2(49-24l)\sqrt{1-l}}{5040}],$$

$$\tilde{r}_2 = v_1^* \bar{b}_2^1 + v_2^* \bar{b}_2^2 + v_3^* \bar{b}_2^3 = \bigcup_{l\in[0,1]} l[\frac{3(16l+9)\sqrt{19l+81}}{5040}$$

$$+ \frac{2(16l+9)^{\frac{3}{2}} + (19l+81)\sqrt{16l+9}}{5040},$$

$$\frac{30(49-24l) + 100\sqrt{49-24l} + 2(49-24l)^{\frac{3}{2}}}{5040}].$$

Step 6: We get

$$d_1^* = d(\tilde{r}_1, r^*) = \int_0^1 \frac{1}{2}[(\frac{(16l+9)\sqrt{19l+81} + 2(16l+9)^{\frac{3}{2}}}{5040} - 1)^2 + (\frac{10(49-24l) + 10\sqrt{(1-l)(49-24l)}}{5040} + \frac{2(49-24l)^{\frac{3}{2}} + 2(49-24l)\sqrt{1-l}}{5040} - 1)^2]dl)^{\frac{1}{2}} = 0.8727$$

$$d_2^* = d(\tilde{r}_2, r^*) = 0.6295$$

$$d_1^- = d(\tilde{r}_1, r^-) = \int_0^1 \frac{1}{2}[(\frac{(16l+9)\sqrt{19l+81}}{5040} + \frac{2(16l+9)^{\frac{3}{2}}}{5040})^2 + (\frac{10(49-24l) + 10\sqrt{1-l)(49-24l)}}{5040} + \frac{2(49-24l)^{\frac{3}{2}} + 2(49-24l)\sqrt{1-l}}{5040})^2]dl)^{\frac{1}{2}} = 0.1546$$

$$d_2^- = d(\tilde{r}_2, r^-) = 0.4423$$

Step 7: Finally, we have

$$CC_1 = \frac{1}{2}(d_1^- + (1 - d_1^*)) = 0.1409 \quad \text{and} \quad CC_2 = 0.4064.$$

Since CC_2 is higher than CC_1, the group selects the solution $\boldsymbol{S}_2$ as the most satisfactory solution for the research planning problem.

4. Conclusions

The paper proposes a general group-making approach to deal with fuzziness of decision maker's preference for alternatives, decision makers' judgment for selection criteria, and decision maker's weighting. The approach aggregates these fuzzy elements into a group decision which is the most acceptable for the group. As the approach supports criteria generation during a group meeting it is very practical. In particular, it is suitable for any types of fuzzy number, in the practice, decision maker can choose a most suitable one for specific decision problems and in specific environment

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