

Recent Trend of Ratings Business in Japan and Improving Proposal for Ratings Forecasting Model

Sakamaki, Yoshikazu

Department of Value and Decision Science Tokyo Institute of Technology Tokyo, 1528550, Japan
megacity@f3.dion.ne.jp

ABSTRACT

The change of economic circumstances in recent years has increased the gaps of credibility among firms, and the necessity to measure the credibility adequately is an important ever-increasing factor. As the result, it is essential for banks to assess the rating evaluation of borrowers appropriately in order to measure the amount of credit risk. This paper reviews preceding studies regarding model building that enables us to explain or forecast the ratings of firms issued from credit-assessment agencies. The paper then explains the features of the ordered logit model in detail that is nowadays used as a ratings evaluation model throughout the world.

Finally, some defect points of these preceding models are pointed out and the improved ordered logit model on the non-compensatory rule that enables to compensate these defect points is proposed. In addition, the appropriateness of the proposed model is verified based on the actual rating data for the companies.

1 INTRODUCTION

Banks are incorporating credit risk evaluation, as it is a very important procedure to measure the level of default probability from the point of macroscopic portfolio strategy. On the other hand, it forms the foundation of a rating system that is nowadays used for self-assessment of major banks and the new BIS Accord in financial administration.

In Japan, financial institutes that pledge to drive credit risk control forward also measure the VaR of the holding portfolio regularly and hence, the management policy is checked up based on it. However, as we verify the element of risk factors in more detail, the evaluation of individual borrowers becomes considerably important. Unless a certain level of fairness or objectivity is compensated when banking facilities evaluate the individual borrowers, it inevitably becomes difficult to decide the organic management policy based on the result of it. That means it plays a very important role in the concept of credit risk control.

When banking institutes calculate the risk-based necessary net asset in proportion to credit risk, they need to know the trend of rating change of each borrower in advance. At that time, a new BIS Accord admits two types of evaluation methods. One method is the standard method where banking institutes can use the ratings of external credit-assessment agencies, whilst the other one is an internal method in which banking institutes can use the ratings that is calculated based on its proprietary system developed by themselves.

With regards to the internal rating method, after the collapse of the 90's bubble economy, as the spread of self-assessment and discussion of a new BIS Accord, it is becoming clear that banking institutes are recognizing the internal rating system as the basic information for credit risk control.

And it becomes essential for a banking institute to assess the rating evaluation of borrowers appropriately in order to measure the amount of credit risk. Therefore, this paper introduces some preceding studies regarding model building that allows the ratings of firms issued from credit-assessment agencies to be explained or forecasted. The paper then explains the features of the ordered logit model in detail that is nowadays used as ratings evaluation model throughout the world.

Moreover, discrepancies in these models are highlighted and an improved ordered logit model is proposed that compensates for these weak points.

2 RELATED WORKS EXPLANATION OF RATINGS MODEL IN THE PRECEDING STUDY

The statistical approach for the rating of firms originated in the 1970's. In the initial study, a linear regression model was mainly used for ratings forecasting. The development of a rating model was then devised by Kaplan and Urwitz (1979). They highlighted the flawed issues contained in the preceding models based on regression analysis or discriminant analysis, and proposed an improved ratings model based on the ordered probit model for the first time. The model developed by them was very sophisticated and they succeeded to hit the rating of 44 among 64 firms without failing and moreover, even the result of ratings of the firms that could not be predicted correctly could be put at least within just one rating before/behind the correct rating.

Here, the ordered logit model is briefly explained and the ordered probit model that is nowadays used widely as ratings forecasting throughout the world is presented. At first, the ordered logit model is built by expanding the normal binominal logit model based on the explanatory variables that classifies the sample data into one of the plural statuses. It can then be used to analyze which

of the statuses the sample data can be classified only when the statuses are ordered.

Here, x_{im} represents the m^{th} ($m=1,2,\dots,M$) explanatory variable of firm i ($i=1,2,\dots,I$) and β_m represents the estimation parameter for x_{im} . Furthermore, we assume that financial health without an error term is given by the summation of the weighted explanatory variables. That is, we define financial health of firm i by

$$z_i = \sum_{m=1}^M \beta_m x_{im} \quad (1)$$

and with the latent variable z_i defined in equation (1), financial health that contains an error term is given by

$$Z_i = z_i + e_i \quad (2)$$

Moreover, we define the threshold by which the ranking of each firm is decided as

$$-\infty = t_0 > t_1 > t_2 > \dots > t_k > \dots > t_K = \infty \quad (3)$$

Supposing the probability of firm i classified in rank s_k can be defined as p_{ik} and distribution function of the error term e_i as F , we can define p_{ik} by

$$p_{ik} = F(\tau_{k-1} - z_i) - F(\tau_k - z_i) \quad (4)$$

Here, z_i is the financial health as defined in equation (1). Moreover, paying attention to the condition of $F(\infty) = 1, F(-\infty) = 0$, we can obtain an ordered logit model if we assume logistic distribution for the distribution function F , and an ordered probit model if we assume normal distribution for F .

Based on equation (3) and (4), p_{ik} in the ordered logit model can be expressed by

$$p_{ik} = \frac{1}{1 + \exp(z_k - \tau_{i-1})} - \frac{1}{1 + \exp(z_k - \tau_i)} \quad (5)$$

On the other hand, under the condition that

$$-\infty = \tau_{-1} < (\tau_0 = 0) < \tau_1 < \tau_2 < \dots < \tau_i < \dots < \tau_{I-1} = \infty$$

rating probability p_{ik} in the ordered probit model can be expressed by

$$p_{ik} = \Phi(\tau_{i-1} - \sum_{m=1}^M \beta_m x_{km}) - \Phi(\tau_{i-2} - \sum_{m=1}^M \beta_m x_{km}) \quad (6)$$

Here, F represents the cumulative distribution function of normal distribution.

Parameter estimation for these models is done as follows. That is, if we assume that the probability of firm i belonging to status s_k as p_{ik} , the likelihood function regarding parameter vector \mathbf{B} and threshold vector \mathbf{t} can be defined by

In this model, we assume statistical independence in financial health of each firm. Here $d_{ik} = 1$ if the financial health of i^{th} firm belongs to s_k and $d_{ik} = 0$ if the financial health of i^{th} firm does not belong to s_k . Moreover, the log-likelihood function can be defined as equation (7), and parameter vector \mathbf{B} and \mathbf{t} that maximizes equation (11) is estimated.

$$l(\mathbf{B}, \mathbf{t} | \mathbf{x}) = \prod_{i=1}^I \prod_{k=1}^K \delta_{ik} \log p_{ik} \quad (7)$$

In a preceding study, Nakayama and Moridaira (1998) studied the verification of parameter estimation accuracy between the ordered logit and ordered probit models, and showed that there are no significant differences of parameter estimation accuracy between the models. On the other hand, Kobayashi (2001) proposes original statistical criteria from his own point of view and verifies that the ordered probit model can be altered as the extreme limit of multivariate probit model.

3 IMPROVEMENT PROPOSAL OF THE TRADITIONAL RATINGS MODEL

As mentioned in Chapter.2, the ordered probit/logit model is built as an expansion of the binominal logit model for classifying sample data into one of the plural statuses. These models are very useful when a certain order is given to the statuses, and it is shown in the preceding studies that these models are possible to be applied in many academic case studies. (For example, when analysts forecast the ratings of firms, credit risk and debt etc.,)

However, the ordered probit model is generally very difficult to use because of statistical reasons. Therefore, the ordered logit model is used mainly in an actual business. In this paper, we focus on the ordered logit model.

The ordered logit model is based on the scores calculated from the summation of weighted variables (linear regression analysis). That is, linearly linked variables are comprehensively evaluated and the rating in which sample data belongs to can be decided. In this sense, this type of model is built on compensatory rule. This type of model can therefore be termed as an "ordered logit model on compensatory rule" in order to distinguish it from our model.

If, in this type of model, a certain explanatory variable happens to take a huge value, the calculated probability can become far from the realistic financial health, and sample data is classified into an extremely superior/inferior rating group.

On the other hand, Katahira *et al.* (1998) have reported a very interesting study in the field of marketing research. In their study, a consumers' choice behavior model is built with the assumption that consumers make a choice behavior based on the compensatory rule. However, as we can see in such a choice behavior, no matter how cheap the merchandise is, consumers do never accept it, unless the quality satisfies a certain criteria. Consumers therefore exhibit behavior based on the non-compensatory rule throughout a variety of occasions.

Concerning this point, Dawes and Corrigan (1974), and Jhonson and Meyer (1984) have shown that under a cer-

tain condition, we can approximate a choice model on the non-compensatory rule by that on the compensatory rule. However, as pointed out in the preceding study where consumers exhibit a choice behavior based on the non-compensatory rule (Bettman and Jacoby (1976), Payne and Ragsdale (1978)), we can not approximate a choice behavior model on the non-compensatory rule by that on the compensatory rule, if a very strong negative correlation between arbitrary two variables exists (Johnson, Meyer and Ghose (1989)).

To overcome such an issue, we will apply the concept of the choice model on the non-compensatory rule to the traditional ordered logit model. Considering the ratings forecasting model of firms, in many cases, variables used as risk factors in the model like capital adequacy ratio and interest-bearing debt have a very strong correlation with each other. Moreover, considering the relationship between the rating of firms and risk factors, there is a tendency that most of the values of risk factors go up or down according to the rating of firm changes. Taking these points into consideration, it seems to be more realistic to apply a rating forecasting model on the non-compensatory rule in which rating changes only when all or most of the risk factors considered in the model goes up/down simultaneously, rather than to apply the model based on the compensatory rule.

>From the study based on the Katahira model, Sakamaki (2003) uses the publicly disclosed financial data of firms and shows that the logit model based on the non-compensatory rule can forecast the default probability more accurately than the normal linearly linked logit model based on the compensatory rule.

When examining the preceding literature, especially in studies where a strongly negative correlation between two explanatory variables exists, it can be thought that it is very meaningful to build a statistical model on the non-compensatory rule rather than the linear model on the compensatory rule. So in this paper, taking these points into consideration, the improved ordered logit model is proposed based on the non-compensatory rule as is shown below.

Here, two types of ordered logit models are proposed based on the non-compensatory rule.

[Conjunctive Type]

In this model, it is assumed that the rating change happens only when all of the risk factors used in the model shift simultaneously to the direction of changing the ratings.

$$\begin{aligned}
 p_{ik} &= p_{ik}(\tau_{mk-1} - x_{im} > \varepsilon \geq \tau_{mk} - x_{im}, \forall m) \\
 &= \prod_{m=1}^M p_{ik}(\tau_{mk-1} - x_{im} > \varepsilon \geq \tau_{mk} - x_{im}) \\
 &= \prod_{m=1}^M \left\{ \left(\frac{1}{1 + \exp(\beta_m(x_{im} - \tau_{mk-1}))} \right) - \left(\frac{1}{1 + \exp(\beta_m(x_{im} - \tau_{mk}))} \right) \right\} \quad (8)
 \end{aligned}$$

[Disjunctive Type]

In this model, it is assumed that the rating change happens when some of the risk factors shift to the direction of changing the ratings.

$$\begin{aligned}
 p_{ik} &= p_{ik}(\exists m, \tau_{mk-1} - x_{im} > \varepsilon \geq \tau_{mk} - x_{im}) \\
 &= 1 - \prod_{m=1}^M p_{ik}(\tau_{mk-1} - x_{im} > \varepsilon \geq \tau_{mk} - x_{im}, \forall m) \\
 &= 1 - \prod_{m=1}^M \left(1 - \left\{ \left(\frac{1}{1 + \exp(\beta_m(x_{im} - \tau_{mk-1}))} \right) - \left(\frac{1}{1 + \exp(\beta_m(x_{im} - \tau_{mk}))} \right) \right\} \right) \quad (9)
 \end{aligned}$$

Here, x_{im} represents the m^{th} ($m=1,2,\dots,M$) explanatory variable of the firm i ($i=1,2,\dots,I$). β_m represents the estimation parameter and τ_{mk} represents the threshold parameter that decides the ratings of each firm in explanatory variable x_{im} . This type of model is termed the "Ordered Logit model On Non-Compensatory Rule" in order to identify it with the normal ordered logit model. Parameter estimation for these models is done as follows. That is, if we assume that the probability of firm i belonging to status s_k as p_{ik} , the likelihood function regarding parameter vector \mathbf{B} and threshold vector \mathbf{t} can be defined by

$$L(\mathbf{B}, \mathbf{t} | \mathbf{x}) = \prod_{i=1}^I \prod_{k=1}^K p_{ik}^{\delta_{ik}} \quad (10).$$

We assume statistical independence in financial health of each firm. Here $d_{ik} = 1$ if the financial health of i^{th} firm belongs to s_k and $d_{ik} = 0$ if the financial health of i^{th} firm does not belong to s_k .

Moreover, the log-likelihood function can be defined as equation (11), and parameter vector \mathbf{B} and \mathbf{t} that maximizes equation (11) is estimated.

$$l(\mathbf{B}, \mathbf{t} | \mathbf{x}) = \prod_{i=1}^I \prod_{k=1}^K \delta_{ik} \log p_{ik} \quad (11)$$

Additionally, in the next chapter, applying publicly disclosed financial data to our model verifies the appropriateness of the model. Moreover, we show the proposed model is superior to the preceding ordered logit model from the view of fitness degree and forecasting power.

4 MODEL VALIDATION

In this chapter, actual data is applied to the model proposed and verifies the appropriateness of the model. The verification data used was publicly disclosed financial data issued from Rating and Investment Inc.

4-1. Outline of verification data

1) Account settlement Term

Rating data that is evaluated and published based on the account settlement data from Apr.,2000 to Mar.,2001

2) Data volume: 652 firms

(Financial data in which missing data does not contain. Moreover, in this study, the power industry is omitted because the relationship of actual management and financial health is very different with that of normal firms)

3) Financial ratio used in the model

Based on the business experience, the following six indexes that can affect the ratings of firms materially are used in the model.

1. log-transformed net asset
2. total capitalization ratio
3. ratio of fixed assets to long-term capital
4. capital adequacy ratio
5. retained earnings ratio
6. dead capacity ratio

4-2 Parameter estimation

Based on this data, parameters of the three types of models, ordered logit model based on the non-compensatory rule (conjunctive/disjunctive type), and the normal ordered logit model are estimated by maximum likelihood estimation.

Generally, the fitness of the model on the training data set is fine, but the most important question is how the model works in out-of-sample data. This is known as generalization, and there are several modeling strategies available to address it. Since the training data set is all the modeler has to work with, strategies to address generalization involve splitting the sample into sub-samples.

The simplest way to split samples is to use two sub-samples randomly selected from the full training data set. The first sub-sample is used as the actual training data set. The second sub-sample, called the *holdout* sample, is not used in the initial estimation, but later, to test the performance of the estimated model. Similar performance on both the training data set and the holdout sample is an indication that the model will perform at about the same level. If the performance of the model deteriorates

substantially on the holdout sample, the model is sensitive to specific aspects of the two sets. Such specifics could include a set of unusual observations, referred to as outliers, which are more prevalent in some data sets. Clusters of observations that show a particular set of relationships that are not always found could also cause performance differences. In estimating parameters, we have chosen 60% of the sample data at random and used for in-sample-data, and 40% for out-of-sample data.

Here, in this study, the classification of ratings is simplified based on Table 1 because there are very few number of firms classified in rating AAA, and we have to consider too many rating patterns if so-called 'notch' described by positive/negative sign is considered in the model.

Table 1. Definition of ratings used in our study

R&I	Rat- ing	Rating in this study	R&I	Rat- ing	Rating in this study
AAA		Over AA	BB+		Under BB
AA+			BB		
AA			BB-		
AA-			B+		
A+		A	B		Under CCC
A			B-		
A-					
BBB+		BBB			
BBB					
BBB-					

Table 2. The number of firms contained in each rating

RATINGS	Number of firms	Ratio
AA	77	11.81%
A	201	30.83%
BBB	269	41.26%
BB	105	16.10%
Total	652	100. 00%

In this study, the parameters are estimated in each model by the best-subset selection method because it is very difficult to find the optimum combination and significant parameters effectively based on the normal variable choice method such as a stepwise algorithm. In the process of parameter estimation, twenty combinations of the initial value are generated for each parameter and the parameters are estimated by the maximum likelihood estimation method based on the initial value. In addition, the combination of parameters that minimizes the AIC and log-likelihood value are adopted as an optimum solution in each model.

The result of parameter estimation is shown in Table 3.

Table 3. Results of parameter estimates

Parameter	Normal Ordered Logit Model	t-value	Non-Compensatory Model (Conjunctive)	t-value	Non-Compensatory Model (Disjunctive)	t-value
β_a	1.782606	(16.145115)	2.304864	(10.782407)	3.102075	(7.513552)
β_b	1.703725	(1.500131)	-	-	-	-
β_c	-1.558375	(-3.405277)	-	-	-	-
β_d	-2.992869	(-2.756496)	-	-	-	-
β_e	8.349215	(8.041677)	11.123563	(10.072548)	20.005778	(8.599619)
β_f	-1.109246	(-4.757166)	-	-	-	-
T_{aa}	35.050708	(15.748833)				
T_a	31.355626	(14.957315)				
T_{bbb}	27.365669	(13.782615)				
$T_{a,aa}$			19.151688	(12.323935)	20.204592	(7.714131)
$T_{a,a}$			17.046034	(11.015636)	18.804618	(7.528707)
$T_{a,bbb}$			16.399593	(10.097468)	16.709588	(7.489304)
$T_{e,aa}$			0.650819	(7.241521)	1.064664	(0.022608)
$T_{e,a}$			0.478657	(9.869452)	0.187891	(7.278373)
$T_{e,bbb}$			0.066425	(0.851711)	0.042501	(3.267568)
Log-Likelihood	-494.490689		-467.330704		-543.979679	
AIC	1006.981379		950.661409		1103.959358	
Hitting ratio	65.48%		68.87%		59.82%	

Note: Based on the estimated parameters, the probability of a firm belonging to each rating is forecasted. If the result of forecasting and the actual rating corresponds perfectly, we regard that as a successful forecast.

The meaning of suffix x in β_x is and $T_{x,y}$ is as follows. $a \rightarrow$ log-transformed net asset, $b \rightarrow$ total capitalization ratio, $c \rightarrow$ ratio of fixed assets to long-term capital, $d \rightarrow$ capital adequacy ratio, $e \rightarrow$ retained earnings ratio, $f \rightarrow$ dead capacity ratio, and suffix y in $T_{x,y}$ means ratings.

5 RESULTS AND DISCUSSION

5.1 Findings

The statistical models studied in the preceding study, especially the ordered logit model has been used in the academic field of credit ratings, consumer's choice behavior etc., as a way to classify sample data into one of the ordered plural statuses. Preceding models are built based on a so-called linearly linked ordered logit model based on the compensatory rule, in which weighted explanatory variables used in the model are linearly linked. As mentioned in the preceding chapter, in the linear model based on the compensatory rule, if there exists a very serious relationship between two particular variables such as capital adequacy ratio and interest-bearing debt, the forecasting power decreases. Moreover, it is likely to happen that if one of the risk factors used in the model has extremely large/small values, a wrong rating evaluation inconsistent with the actual management status will be given to the firms.

Paying attention to each risk factor in the preceding model, it was difficult to know the threshold value among the ratings by each factor. Considering these points, an ordered logit model is proposed based on the

non-compensatory rule in order to overcome these issues in this paper, and the appropriateness of the model is verified by applying actual data to it.

It can be shown that new model proposed in this study can improve all of the log-likelihood, AIC criteria, and the hitting ratio of rating forecasting compared with the ordered logit model. Moreover, regarding the risk factor used in the model, we used all of the risk factors available in the model to optimize the fitness in the preceding model. Whilst on the other hand, it could be shown that we could get a higher fitting degree by using only two kinds of risk factors in the proposed model. The reason for this can be thought to be that in the preceding model, many kinds of risk factors that have a strong correlation to each other are contained in the model. This brings an adverse affect to the model, which as a result, leads to a declined forecasting power.

Furthermore, the result of parameter estimation is considered. Comparing the result of parameters estimated in each model, whilst regarding the identical parameter, the size of each parameter seems to vary among the models. However, the positive/negative sign of the identical parameter matches perfectly. It can therefore be assumed that the parameter estimation was performed accurately.

5-2 Future Investigations

Finally, there are issues of this study that require improvement for future work. Generally, when we estimate the parameters in the normal logit model, we can estimate the parameters effectively by using a parameter estimation algorithm such as a stepwise method. However, in this proposed model, the following point still remains where we have to assume two kinds of parameters for one explanatory variable, so that it is impossible to apply the normal stepwise algorithm to the non-compensatory type of model.

This study has adopted the best-subset selection method as the parameter estimation algorithm. However, this method does not always choose the optimal combination patterns of parameters of the model. Moreover, the number of combination patterns of variables increases rapidly and the length of time needed to find the optimum value of the parameters increases as the number of variables used in the model increases.

Regarding our model on the non-compensatory rule, applicability of the model is reported to be very various in the preceding study. For example, it can be used not only for ratings model but also for consumer's choice behavior model in marketing research and traveler's decision making for transformation method in social engineering science, and so on. Academic fields to which statistical model on non-compensatory rule can be applied, should spread rapidly in the near future. Variable choice algorithms such as stepwise, backward, and forward methods are proposed in the study of a linear model based on the compensatory rule. We expect the same type of variable choice algorithm that enables the analysts to use the model easily should be developed in the future.

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AUTHORS BIOGRAPHIES

Yoshikazu, Sakamaki is a student of doctor course in Tokyo Institute of Technology, Department of Value and Decision Science, Japan. He received a B.Eng in the Department of Control and System Engineering from Tokyo Institute of Technology and his M.Eng in the Department of Industrial Engineering and Management from Tokyo Institute of Technology. His recent work has involved financial engineering, marketing research with use of statistical approach. His email address is megacity@f3.dion.ne.jp.