

Credibility Theory: A New Branch of Mathematics that Studies the Behavior of Fuzzy Phenomena

Baoding Liu

Uncertainty Theory Laboratory, Department of Mathematical Sciences

Tsinghua University, Beijing 100084, China

liu@tsinghua.edu.cn

http://orsc.edu.cn/~liu

Abstract: Fuzzy set was initialized by Zadeh via membership function in 1965, and was well developed and applied in a wide variety of real problems. In order to measure a fuzzy event, Zadeh proposed the concept of possibility measure in 1978. Although possibility measure has been widely used, it has no self-duality property. However, a self-dual measure is absolutely needed in both theory and practice. In order to define a self-dual measure, Liu and Liu presented the concept of credibility measure in 2002. Credibility theory is a branch of mathematics that studies the behavior of fuzzy phenomena. An axiomatic foundation of credibility theory was given by Liu in 2004. In fact, the whole credibility theory was derived from the following five axioms:

Axiom 1. $\text{Cr}\{\Theta\} = 1$.

Axiom 2. $\text{Cr}\{A\} \leq \text{Cr}\{B\}$ whenever $A \subset B$.

Axiom 3. Cr is self-dual, i.e., $\text{Cr}\{A\} + \text{Cr}\{A^c\} = 1$ for any $A \in \mathcal{P}(\Theta)$.

Axiom 4. $\text{Cr}\{\cup_i A_i\} \wedge 0.5 = \sup_i \text{Cr}\{A_i\}$ for any $\{A_i\}$ with $\text{Cr}\{A_i\} \leq 0.5$.

Axiom 5. Let Θ_k be nonempty sets on which Cr_k satisfy the first four axioms, $k = 1, 2, \dots, n$, respectively, and $\Theta = \Theta_1 \times \Theta_2 \times \dots \times \Theta_n$. Then

$$\text{Cr}\{A\} = \begin{cases} \sup_{(\theta_1, \theta_2, \dots, \theta_n) \in A} \min_{1 \leq k \leq n} \text{Cr}_k\{\theta_k\}, \\ \quad \text{if } \sup_{(\theta_1, \theta_2, \dots, \theta_n) \in A} \min_{1 \leq k \leq n} \text{Cr}_k\{\theta_k\} < 0.5 \\ 1 - \sup_{(\theta_1, \theta_2, \dots, \theta_n) \in A^c} \min_{1 \leq k \leq n} \text{Cr}_k\{\theta_k\}, \\ \quad \text{if } \sup_{(\theta_1, \theta_2, \dots, \theta_n) \in A} \min_{1 \leq k \leq n} \text{Cr}_k\{\theta_k\} \geq 0.5 \end{cases}$$

for each $A \in \mathcal{P}(\Theta)$. In that case we write $\text{Cr} = \text{Cr}_1 \wedge \text{Cr}_2 \wedge \dots \wedge \text{Cr}_n$.

Let Θ be a nonempty set, $\mathcal{P}(\Theta)$ the power set of Θ . The set function Cr is called a credibility measure if it

satisfies the first four axioms. The triplet $(\Theta, \mathcal{P}(\Theta), \text{Cr})$ is called a credibility space.

A fuzzy variable is defined as a function from a credibility space $(\Theta, \mathcal{P}(\Theta), \text{Cr})$ to the set of real numbers. Then its membership function is derived from the credibility measure by

$$\mu(x) = (2\text{Cr}\{\xi = x\}) \wedge 1, \quad x \in \mathfrak{R}.$$

Conversely, let ξ be a fuzzy variable with membership function μ . Then for any set B of real numbers, we have

$$\text{Cr}\{\xi \in B\} = \frac{1}{2} \left(\sup_{x \in B} \mu(x) + 1 - \sup_{x \in B^c} \mu(x) \right).$$

The expected value of ξ is defined by

$$E[\xi] = \int_0^{+\infty} \text{Cr}\{\xi \geq r\} dr - \int_{-\infty}^0 \text{Cr}\{\xi \leq r\} dr$$

provided that at least one of the two integrals is finite.

This talk will introduce the concepts of credibility measure, credibility distribution, expected value, variance, entropy, characteristic function, convergence almost surely, convergence in credibility, convergence in mean, convergence in distribution, and show the credibility subadditivity theorem, credibility extension theorem, credibility semicontinuity law, sufficient and necessary condition for credibility distribution, linearity of expected value operator, inversion formula. The interested reader may download the book from <http://orsc.edu.cn/~liu>.

Keywords: fuzzy variable, credibility measure, axioms.

Baoding Liu has been a full Professor at Tsinghua University since 1998. He graduated from the Nankai University in 1986, and received the M.S. and Ph.D. degrees from Chinese Academy of Sciences in 1989 and 1993, respectively. His current research interests include uncertainty theory, uncertain programming, intelligent algorithm and applications. He is the author of 7 books, including *Uncertainty Theory* (Springer), *Theory*

and Practice of Uncertain Programming (Physica), *Decision Criteria and Optimal Inventory Processes* (Kluwer), and *Uncertain Programming* (Wiley). Dr. Liu serves on the editorial boards of 6 international journals, including IEEE Transactions on Fuzzy Systems, Information: An

International Journal, Fuzzy Optimization and Decision Making, Asian Information-Science-Life, International Journal of Internet and Enterprise Management, and International Journal of Operations Research.