

## Research on promotion mode of dual channel supply chain considering consumer channel preference

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### ABSTRACT

This paper concentrates on the dual-channel supply chain considering consumer channel preference, which manufacturers through online direct sales and retailers through offline retail, and constructs different promotion models: manufacturers and retailers do not promote, the retailer does promotion and the manufacturer does promotion, while considering consumers' channel preferences, study the impact of promotion on the profit and performance of supply chain system members, and finally get the best promotion strategy for retailers and manufacturers through comparative analysis. It is of extraordinary viable importance to make reasonable promotion decisions for members of the dual-channel supply chain system.

*Keywords:* Dual channels, supply chain, promotion effort, channel preference.

### 1. INTRODUCTION

With the development of the Internet, product information has become more open and transparent, and the homogeneity of products sold by companies has become more and more serious. "Promotion" has become an important means for supply chain companies to improve their own performance. "Promotion" refers to a way for companies to use various activities and methods to attract consumers to pay attention to their products, stimulate consumers' desire to buy, and promote product sales. This article refers to the act of "promotion" by members of the supply chain as "promotional effort". Offline retailers carry out promotions through shop display board advertisements, leaflet printing, and human sales promotion; online merchants rely on the advantages of a large number of Internet users to promote sales through paid personalized push, full reduction activities and other promotional methods. As the advantages of the Internet have become increasingly prominent, manufacturers have gradually opened up their own online direct sales channels, forming a dual-channel supply chain where both online and offline coexist. According to the different promoters of the promotion, this article divides it into three different models, namely, the manufacturer and the retailer do not make promotion efforts, the retailer side makes the promotion effort, and the manufacturer side makes the promotion effort.

So what impact do these three promotion efforts models have on the profits and performance of the members of the dual-channel supply chain system? Huang and Bai believe that the promotional reference effect will lessen the benefits of retailers and increase the profits of manufacturers (Huang et. al, 2018). Li and Yang believe that retailers always charge higher prices when they carry out coupon promotions (Li et. al, 2021). Jin establish a supply chain model with limited capital and demand dependent on retail price and promotion efforts, and find that the chain operation mode is beneficial to all supply chain members (Jin et.al, 2015). Heidarpour and Heydari believe that the buy-one-get-one incentive scheme will make the supply chain more profitable under the coordination contract (Heydari et.al, 2020). Bai and Chen believe that in the supply chain where demand is affected by promotional efforts, the cooperation between manufacturers and retailers can generate higher profits (Bai et.al, 2017). Tsay and Agrawa considers the impact of channel inclination and sales effort on demand, the dual channel pricing strategy (Tsay & Agrawal, 2009).

From the current research literature, it can be found that there are few studies that combine consumer channel preferences and dual-channel supply chain promotion models at the same time. In order to be close to the reality, this article also takes into account the consumer's channel preferences, establishes a manufacturer-led Stackelberg game model, compares and analyzes the optimal decision and profit of the manufacturer and the retailer, and obtains the optimal promotion for the manufacturer and the retailer. Strategies have important reference value and practical significance for the development of promotion strategies for dual-channel supply chain companies.

### 2. PROBLEM DESCRIPTION AND MODEL BUILDING

#### 2.1. Problem Description

This article studies a dual-channel supply chain composed of manufacturers' online direct deals channels and retailers' disconnected retail channels, in which manufacturers wholesale products to retailers at a unit price  $w$ , and retailers sell to consumers at a unit price  $P_r$ . Manufacturers of online direct sales channels sell products to consumers at a unit price  $P_m$ . According to the different promoters of the promotion, it can be divided into the following modes, as shown in Figure 1..

Mode N: Manufacturers and retailers do not make promotions

Mode R: The retailer does a promotion  
 Mode M: The producer does a promotion

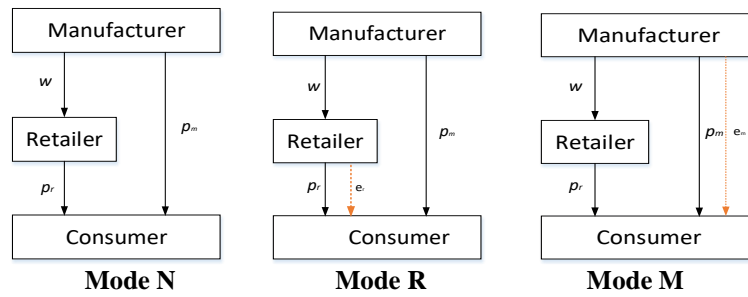


Figure 1. Diagram of the promotion effort model of the dual-channel supply chain system

## 2.2. Model building

According to the above three modes, the profit function of the system members is established, and then the respective optimal decisions are solved, and the profit situation under the various modes is compared and analyzed, and the optimal promotion strategy is obtained. In the following text, the superscripts N, R, and M are used to denote the N mode, R mode and M mode, respectively, and the superscript \* denotes the optimal decision. The subscripts r and m denote the retailer and the manufacturer, respectively, and  $\pi_r$  and  $\pi_m$  denote the retailer, respectively. And the profit of the manufacturer, the total profit of the supply chain system is expressed as  $\pi$ , and  $\pi = \pi_r + \pi_m$ .

The basic hypothesis of this article : ① The production cost of the manufacturer will not have a substantial impact on the research results of this article. It is assumed that the creation cost of the maker is zero. ② Manufacturers and retailers are completely rational and risk-neutral, and both take the maximization of their own interests as their decision-making goals. ③ The level of promotion effort can be accurately quantified. ④ Referring to the product demand function constructed by Huo Hong [3] and Liu Xinmin [7], market demand is affected by the online direct selling price  $p_m$ , offline retail price  $p_r$ , the influence coefficient  $\beta$  ( $\beta > 0$ ) of promotion effort on demand, and promotion The impact of effort level  $e_i$  ( $e_i > 0$ ), where  $i=r, m$  represent the retailer and the manufacturer, respectively. Suppose the offline market demand function is  $q_r = na - p_r + bp_m + \beta e_r$ , and the online market demand function is  $q_m = (1 - n)a - p_m + bp_r + \beta e_m$ . Among them, a represents the potential market demand, n represents the consumer's preference for consumption in traditional retail channels ( $0 < n < 1$ ), and b represents the consumer's price sensitivity coefficient ( $0 < b < 1$ ), which can reflect consumers' competitive price The degree of sensitivity. ⑤ Similar to the research of Huo Hong [10], the promotion effort cost is set as  $C(e_i) = 1/2 \mu e_i^2$ , where  $\mu$  ( $\mu > 0$ ) describes the cost coefficient of promotion effort. ⑥ The manufacturer is the head of the supply chain, and the retailer is the adherent.. Both of them obey the Stackelberg game in their decision-making..

## 3. MODEL OPTIMAL DECISION

### 3.1. Model N: Manufacturers and retailers do not make promotions

This section studies the N mode as a benchmark for comparison of several other modes. When neither the manufacturer nor the retailer makes promotional efforts,  $e_r = 0$  and  $e_m = 0$  at this time. The demand functions of retailers and manufacturers are as follows:

$$q_r = na - p_r + bp_m, q_m = (1 - n)a - p_m + bp_r$$

The profit functions of retailers and manufacturers as per the following:

$$\pi_r = (p_r - w)q_r, \pi_m = p_m q_m + wq_r$$

Next, as indicated by the profit function of the manufacturer and the retailer in the N model, the manufacturer's optimal wholesale price and the optimal online direct selling price can be obtained; the retailer's optimal retail price; the manufacturer and the retailer's respective maximum profit .

*Theorem 1:* In model N, the optimal decision and profit of the supply chain are as follows:

$$w^{N*} = \frac{a(n+b-nb)}{2(1-b^2)}, p_m^{N*} = \frac{a(1-n+nb)}{2(1-b^2)}, p_r^{N*} = \frac{a(2b+3n-2nb-nb^2)}{4(1-b^2)}$$

$$\pi_m^{N*} = \frac{a^2(n^2b^2-4n^2b+4nb+3n^2-4n+2)}{8(1-b^2)}, \pi_r^{N*} = \frac{n^2a^2}{16}$$

Proof: It can be obtained by using reverse derivation method, See appendix for proof.

### 3.2. Mode R: Only retailers make promotion decisions

When only the retailer makes promotion efforts,  $e_r > 0$  and  $e_m = 0$  at this time. The demand functions of retailers and manufacturers are as follows:

$$q_r = na - p_r + bp_m + \beta e_r, \quad q_m = (1 - n)a - p_m + bp_r$$

The profit functions of retailers and manufacturers are as follows:

$$\pi_r = (p_r - w)q_r - \frac{1}{2}\mu e_r^2, \quad \pi_m = p_m q_m + wq_r$$

Next, as indicated by the profit function of the manufacturer and the retailer in the R model, the manufacturer's optimal wholesale price and the optimal online direct selling price can be obtained; the retailer's optimal retail price; the manufacturer and the retailer's respective maximum profit And the total profit of the dual-channel supply chain system

*Theorem 2:* In model R, the optimal decision and profit of the supply chain are as follows

$$\begin{aligned} w^{R*} &= \frac{a(4\mu^2(b+n-bn)-4b\mu\beta^2(1-n)-n\mu\beta^2(2+b^2)+b\beta^2(1-n\beta^2))}{4\mu(1-b^2)(2\mu-\beta^2)-\beta^4b^2}, p_m^{R*} = \frac{a\mu(2(1-n)(2\mu-\beta^2)+bn(4\mu-\beta^2))}{4\mu(1-b^2)(2\mu-\beta^2)-\beta^4b^2} \\ p_r^{R*} &= \frac{a((1-n)(4b\mu^2-3\beta^2b\mu+\beta^4b)+2n\mu(\mu(3-b^2)-\beta^2))}{4\mu(1-b^2)(2\mu-\beta^2)-\beta^4b^2}, e_r^{R*} = \frac{a\beta(2n\mu(1-b^2)+b\beta^2(1-n+bn))}{4\mu(1-b^2)(2\mu-\beta^2)-\beta^4b^2} \\ \pi_m^{R*} &= \frac{a^2\mu(\beta^2(n(bn-b-n+2)-1)-4\mu n(bn+1-b)+\mu(b^2n^2+3n^2+2))}{4\mu(1-b^2)(2\mu-\beta^2)-\beta^4b^2}, \pi_r^{R*} = \frac{a^2\mu(2\mu-\beta^2)(2n\mu(1-b^2)+b\beta^2(1+bn-n))^2}{2(4\mu(1-b^2)(2\mu-\beta^2)-\beta^4b^2)^2} \end{aligned}$$

Proof: It can be obtained by using reverse derivation method, See appendix for proof.

### 3.3. Mode M: Only the manufacturer makes promotion decisions

Only when the manufacturer makes promotional efforts,  $e_r=0$  and  $e_m>0$  at this time. The demand functions of retailers and manufacturers are as follows:

$$q_r = na - p_r + bp_m, \quad q_m = (1 - n)a - p_m + bp_r + \beta e_m$$

The profit functions of retailers and manufacturers are as follows:

$$\pi_r = (p_r - w)q_r, \quad \pi_m = p_m q_m + wq_r - \frac{1}{2}\mu e_m^2$$

Next, according to the profit function of the manufacturer and the retailer in the model M, the manufacturer's optimal wholesale price and the optimal online direct selling price can be obtained; the retailer's optimal retail price; the manufacturer and the retailer's respective maximum profit.

*Theorem 3:* In model M, the optimal decision and profit of the supply chain are as follows:

$$\begin{aligned} w^{M*} &= \frac{a(2\mu(b+n-bn)-n\beta^2)}{2(2\mu-\beta^2-2\mu b^2)}, p_m^{M*} = \frac{a\mu(1+bn-n)}{2\mu-\beta^2-2\mu b^2}, e_m^{R*} = \frac{a\beta(1+bn-n)}{2\mu-\beta^2-2\mu b^2}, p_r^{M*} = \frac{a(2b\mu(2-bn-2n)+3n(2\mu-\beta^2))}{4(2\mu-\beta^2-2\mu b^2)}, \\ \pi_m^{M*} &= \frac{a^2(8\mu n(b-1-bn)+n^2(6\mu+2\mu b^2-\beta^2)+4\mu)}{8(2\mu-\beta^2-2\mu b^2)}, \pi_r^{M*} = \frac{a^2n^2}{16} \end{aligned}$$

Proof: It can be obtained by using reverse derivation method, See appendix for proof.

## 4. MODEL COMPARISON ANALYSIS

### 4.1. Comparative Analysis of Optimal Decisions

*Proposition 1:* In model N, model R, and model M, the traditional retail channel demand  $q_r$  and retailer's promotion effort level  $e_r$  are increasing functions of  $n$ , and the direct sales channel demand  $q_m$  and the manufacturer's promotion effort level  $e_m$  are decreasing functions of  $n$ .

Proof: It is easy to obtain the derivative of each of the above quantities with respect to  $n$ , which is omitted here.

*Proposition 2:*(1) When the retailer makes promotional efforts, the demand for offline retail channels increases, that is,  $q_r^{R*} > q_r^{M*} = q_r^{N*}$ .(2) When the manufacturer makes sales promotion efforts, the demand for online direct sales channels increases, and the demand for offline retail channels decreases, that is,  $q_m^{M*} > q_m^{N*} > q_m^{R*}$ .

Proof: It can be obtained by making the difference of the demand in the proposition compared with each other, which is omitted here.

*Proposition 3:* The manufacturer's online direct selling price  $p_m$  is a decreasing function of  $n$ , while the manufacturer's wholesale price  $w$  and the retailer's retail price  $p_r$  are an increasing function of  $n$ .

Proof: It can be obtained by deriving the derivation of each quantity in the proposition with respect to  $n$ , which is omitted here.

*Proposition 4:* When the retailer makes a promotional effort, the manufacturer's wholesale price will decrease, that is,  $w^{R^*} < w^{N^*}$ ; when the manufacturer makes a promotional effort, the manufacturer's wholesale price will be increase, that is,  $w^{M^*} > w^{N^*}$ .

Proof: The wholesale price of the manufacturers compared with each other in the proposition can be obtained by making a difference, which is omitted here.

Proposition 4 shows that when a retailer makes a promotion effort, the retailer will pay the cost of the promotion effort. In order to alleviate the "double marginal effect", the manufacturer will lower the wholesale price. When a manufacturer makes a promotion effort, the manufacturer will pay the cost of the promotion effort. In order to maintain its own income and improve its competitive advantage, the manufacturer increases the wholesale price to the retailer.

*Proposition 5:* When a manufacturer or retailer makes a promotional effort:

On the one hand, offline retail prices of retailers will rise, that is,  $p_r^{R^*} > p_r^{N^*}$ ,  $p_r^{M^*} > p_r^{N^*}$ ;

On the other hand, the direct selling prices of manufacturers will also increase, that is,  $p_m^{R^*} > p_m^{N^*}$ ,  $p_m^{M^*} > p_m^{N^*}$ .

Proof: The difference between the retailer's retail price and the manufacturer's online direct selling price in the proposition can be obtained, which is omitted here.

Proposition 5 shows that when retailers make promotional efforts, consumer demand in traditional retail channels increases. In order to obtain more profits, retailers raise retail prices at this time, but they use promotional methods to feed back some benefits. Because the products of manufacturers and retailers are competitive between channels, to increment online profits, manufacturers also appropriately increase online direct selling prices. When the manufacturer makes a sales promotion effort, the demand for online direct sales channels increases, and the manufacturer increases the direct sales price in order to obtain more profits.

## 4.2. Maximum profit comparative analysis

*Theorem 4:* As long as the retailer makes promotional efforts, the retailer's profit will increase, and the retailer will profit from it, that is,  $\Pi_r^{R^*} > \Pi_r^{M^*} = \Pi_r^{N^*}$ .

Proof: The profit of each retailer compared with each other in the proposition can be obtained by making a difference, which is omitted here.

Theorem 4 shows that when a retailer makes a marketing effort alone, the retailer's profit will increase. This is mainly due to the fact that after retailers have made promotional efforts, the increase in profits brought about by increased retail channel demand, lower wholesale prices and rising retail prices is greater than the reduction in costs of promotional efforts.

*Theorem 5:* (1) As long as the manufacturer or retailer makes promotion efforts, the profit of the manufacturer will increase, that is,  $\Pi_m^{R^*} > \Pi_m^{N^*}$ ,  $\Pi_m^{M^*} > \Pi_m^{N^*}$ .

(2) When  $n < h$ ,  $\Pi_m^{M^*} > \Pi_m^{R^*}$ ; When  $n > h$ ,  $\Pi_m^{M^*} < \Pi_m^{R^*}$ ;

$$\text{among them : } h = \frac{-(8\beta^2\mu + 8b\mu^2 + 2(\mu(2+b^2)(\beta^2 + 2\mu b^2 - 2\mu)(\beta^4 b^2 - 4\beta^2 b^2 + 4\beta^2\mu + 8b^2\mu^2 - 8\mu^2))^{\frac{1}{2}}}{(\beta^4 b^2 - 16b\mu^2 - \beta^2(2\mu b^4 - 8\mu b^3 + 10\mu b^2 - 8\mu b + 4\mu) + 8\mu^2 + 16b^2\mu^2 - 16b^3\mu^2 + 8b^4\mu^2)}$$

Proof: It can be obtained by comparing the profits of each manufacturer in the proposition with each other, which is omitted here.

Theorem 5 shows that whether it is a sales promotion effort by a manufacturer or a retailer, the profit of the manufacturer will increase. When the manufacturer makes promotional efforts, the original consumer demand of the retail channel will not change. The increase in the profit of the manufacturer is due to the increase in the wholesale price of the retail channel on the one hand; on the other hand, the increase in demand for the online direct sales channel and The increase in profits brought about by the increase in direct selling prices is greater than the decrease in the cost of promotion. When the retailer makes a promotional effort, the demand for online direct sales channels will not be affected. The increase in the profit of the manufacturer is due to the increase in retail channel demand, which is greater than the decrease in wholesale price. If only one side of the manufacturer or retailer is considered for promotion efforts, when consumers have less consumer preference for traditional channels ( $n < h$ ), the profit of the manufacturer's independent promotion efforts is greater, which is more beneficial to the manufacturer. When consumers have greater preference for traditional consumption ( $n > h$ ), and retailers make promotional efforts alone, the manufacturer's profits will be greater and it will be more beneficial to the manufacturer.

In order to prove the correctness of the propositions and theorems, the following analysis and verification are carried out by means of numerical analysis.

## 5. NUMERICAL ANALYSIS

Considering that consumers have different consumption preferences for different channels, take consumer preference  $n$  for traditional channels as an independent variable to verify the correctness of the results of this paper. Suppose  $a=800$ ,  $b=0.2$ ,  $\beta=3$ ,  $\mu=6$ .

As shown in Table 1, as consumers' preference  $n$  for offline channels increases, wholesale prices, offline retail prices, and retailers' efforts to make promotions  $e_r$ , all increase, and online direct sales prices and manufacturers' promotion efforts. The degree of effort  $e_m$  at time decreases accordingly, which verifies Proposition 1 and Proposition 3. The wholesale price when the retailer makes a promotion effort is reduced, and the wholesale price when the manufacturer makes the promotion effort is increased, thus verifying Proposition 4. After the manufacturer or retailer made the decision of promotion efforts, offline retail prices have risen, and the increase rate when the retailer made the promotion decision was higher than the situation when the manufacturer made the promotion decision; on the other hand, the line The price of direct sales has also risen, and the increase when the manufacturer makes the promotion decision is higher than the increase when the retailer makes the promotion decision, thus verifying Proposition 5. Studying the demand situation of different channels, we can realize that the retailer's promotion efforts will increase the demand for offline retail channels, but will not affect the original consumer demand of the online direct sales channel; the manufacturer's promotion efforts will increase the demand for online direct sales channels, while the offline retail channels are less affected, and the demand has decreased, thus verifying Proposition 2.

Table 1 Optimal decisions under different channel preference

$n$	Mode	$w$	$p_r$	$p_m$	$q_r$	$q_m$	$e_r$	$e_m$
0.3	$N$	183.33	243.33	316.67	60	292	-	-
	$R$	138.58	490.05	371.58	351.48	286.43	175.73	-
	$M$	409.52	469.52	1447.6	60	1377.7	-	723.80
0.5	$N$	250	350	250	100	220	-	-
	$R$	216.39	528.57	327.87	498.36	215.08	249.18	-
	$M$	428.57	714.75	1142.9	100	1077.1	-	571.42
0.8	$N$	350	510	150	160	112	-	-
	$R$	333.11	1051.8	262.30	718.69	108.07	359.34	-
	$M$	457.14	617.14	685.71	160	626.29	-	342.86

### 5.1. The influence of consumer channel preference on total supply chain demand

It very well may be seen from Figure 2 that as long as a company in the supply chain makes a promotion decision, the overall performance of the system will improve. When  $n \leq 0.78$ , the promotion decision made by the manufacturer can obtain more consumer demand. At this time, online channels are more attractive to consumers, and the performance of the supply chain system is higher. When  $n > 0.78$ , the retailer's decision to make promotion efforts can obtain more consumer demand, because the retailer is closer to the consumer and can provide more in line with expectations. At this time, the traditional retail channel is more for consumers. Attractive, higher performance of the supply chain system.

### 5.2. The influence of consumer channel preference on manufacturers' profits

As per Figure 3, as long as the companies in the supply chain system make promotional efforts, the profits of manufacturers will increase. If only one side of the manufacturer or retailer is considered for promotion efforts, it can be known that when  $n \leq 0.64$ , online channels are more attractive to consumers at this time, and the manufacturer's profit for promotion is higher; when  $n > 0.64$  At this time, consumers prefer to buy products offline. For manufacturers, it is more beneficial for the retailer to make sales promotion efforts. At this time, the manufacturer does not need to make additional promotion efforts, which verifies Theorem 5.

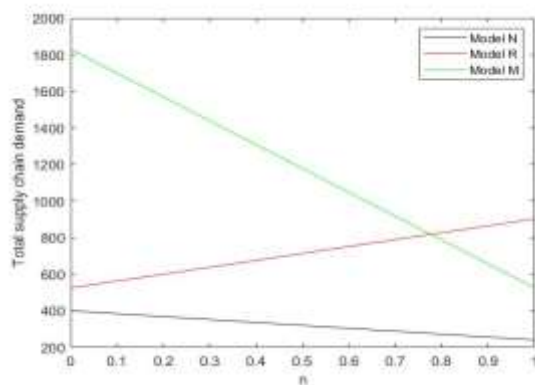


Figure 2. The impact of  $n$  on supply chain system demand

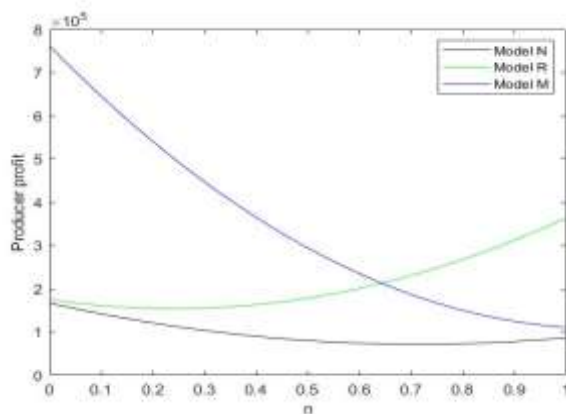


Figure 3. The impact of  $n$  on the profits of manufacturers

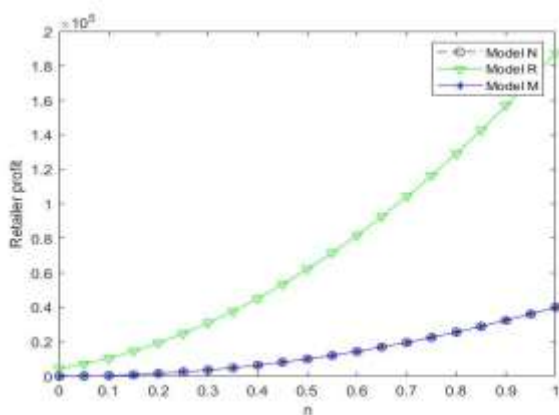


Figure 4. The impact of  $n$  on retailer profits

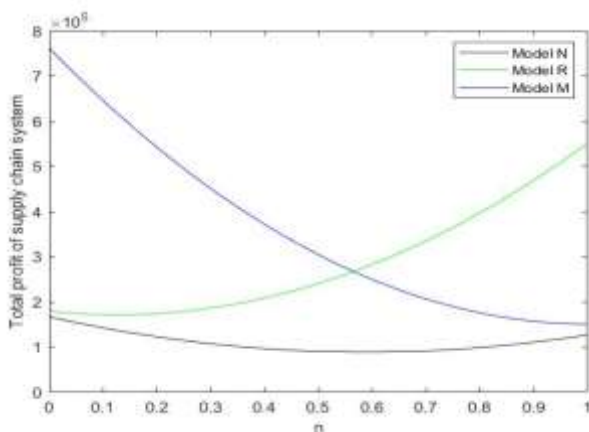


Figure 5. The impact of  $n$  on the profit of the supply chain system



### 5.3. The influence of consumer channel preference on retailer's profit

According to Figure 4, the retailer's profit always increases with the increase of consumers' preference for offline channels. When the manufacturer makes promotion efforts, it has little effect on the retailer's profit. When the retailer makes a promotional effort, the retailer's profits will increase. Therefore, the retailer has the tendency to make the promotion effort decision, which verifies Theorem 4.

### 5.4. The influence of consumer channel preference on the total profit of the supply chain

According to Figure 5, when a company member in the supply chain makes a promotion decision, the total profit of the system will increase significantly. If only the manufacturer or retailer is considered for promotion efforts, when  $n \leq 0.56$ , online channels are more attractive to consumers at this time, and the total profit of the supply chain where the manufacturer makes the promotion effort decision is greater. When  $n > 0.56$ , offline channels are more attractive to consumers at this time, and the total profit of the supply chain where the retailer makes the promotion effort decision is greater.

## 6. CONCLUSION

This paper concentrates on the effect of different promotion efforts on the profit and performance of the members of the dual-channel supply chain system. According to the difference between online and offline promotion initiators, there are three different models: manufacturers and retailers do not promote sales, retailers do sales alone, and manufacturers do sales alone. According to the Stackelberg game method, the optimal decision and optimal profit of the manufacturer and retailer under different promotion modes are obtained respectively, and the optimal decision and profit of the manufacturer and the retailer are compared and analyzed, and the optimal decision of the manufacturer and the retailer is obtained. Promotion strategy has important reference value and practical significance for the development of promotion strategy of dual-channel supply chain enterprises.

The conclusions of this paper are as follows: (1) Retailers have promotion motives: When a retailer makes a promotion effort decision, the manufacturer will lessen the wholesale price in order to alleviate the "double marginal effect". (2) For the manufacturer, the profit will increase if the manufacturer or retailer makes the decision of promotion efforts. (3) When online channels are more attractive to consumers, for manufacturers, their decision-making on promotion efforts will be more profitable, and at the same time, the supply chain performance level will be higher. (4) When traditional channels are more attractive to consumers, as far as manufacturers are concerned, the retailer's promotion efforts can make them more profitable, simultaneously, the performance level of the supply chain will be higher.

In addition, it is possible to further study the situation when manufacturers and retailers make promotional efforts at the same time, considering the asymmetry of demand information and random consumer demand and other special circumstances, the research on the profit and performance of the members of the dual-channel supply chain. This is also a direction worth studying in the future.

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## APPENDIX

**Proof of Theorem 1**

The decision sequence of mode N is that the manufacturer first sets its own wholesale price  $w$  and the online direct selling price  $p_m$ , and then the retailer sets its own retail price  $p_r$  according to the manufacturer's decision.

Use the reverse derivation method to find the optimal solution:

First, the second derivative of the retailer's profit function  $\Pi_r$  to  $p_r$  is  $-2$ , which shows that  $\Pi_r$  is a concave function and has a unique maximum value. Find the first derivative of  $\Pi_r$  and set it to 0 to obtain:  $p_r = \frac{1}{2}(w + na + bp_m)$ . Incorporate the above formula into the producer profit function  $\Pi_m$ , and obtain the Hessian determinant of  $\Pi_m$  with respect to  $w$  and  $p_m$ :  $H(\Pi_m) = \begin{vmatrix} b^2 - 2 & b \\ b & -1 \end{vmatrix}$ . From the formula, the first-order principal and sub-determinant can be obtained  $H(\Pi_m)_{11} = b^2 - 2 < 0$ , the second-order principal and sub-determinant  $H(\Pi_m)_{22} = 2(1 - b^2) > 0$ , it is known that the profit function  $\Pi_m$  is a concave function of  $w$  and  $p_m$ , and there is a unique maximum value. Next,  $\Pi_m$  is differentiated with respect to  $w$  and  $p_m$ , and the parallel equations are solved to obtain  $w^{N*}$  and  $p_m^{N*}$ . From this, theorem 1 in the text can be obtained.

**Proof of theorem 2**

The decision-making sequence of R mode: the manufacturer first sets its own wholesale price  $w$  and online direct selling price  $p_m$ , and the retailer then sets its own retail price  $p_r$  and effort level  $e_r$  according to the manufacturer's decision.

The optimal solution is obtained by inverse derivation:

Assume that the wholesale price  $w$  and the online direct selling price  $p_m$  are known, and then find the retailer profit function  $\Pi_r$  Hessian determinant of retail price  $p_r$  and effort level  $e_r$ :

$$H(\Pi_r) = \begin{vmatrix} -2 & \beta \\ \beta & \mu \end{vmatrix}$$

From the above expression, we can obtain the determinant  $h$  of the first-order principal sub form  $H(\Pi_r)_{11} = -2 < 0$ , the determinant of the second-order principal sub form  $H(\Pi_r)_{22} = 2\mu - \beta^2$ . Retailer profit function  $\Pi_r$  must satisfy  $2\mu - \beta^2 > 0$  if there is an optimal solution. The first derivative of R with respect to  $p_r$  and  $e_r$  is set to zero, and the simultaneous equations are solved to obtain:

$$p_r = \frac{-w\beta^2 + \mu w + na\mu + b\mu p_m}{2\mu - \beta^2}$$

$$e_r = \frac{na\beta - w\beta + b\beta p_m}{2\mu - \beta^2}$$

Substitute the above formula into the profit function  $\Pi_m$ . Available  $\Pi_m$  is the Hessian determinant of  $p_m$  and  $w$ :

$$H(\Pi_m) = \begin{vmatrix} \frac{2\mu b^2}{2\mu - \beta^2} - 2 & b \\ b & \frac{-2\mu}{2\mu - \beta^2} \end{vmatrix}$$

From the above expression, we can obtain the determinant of the first-order principal sub form  $H(\Pi_m)_{11} = \frac{2\mu b^2}{2\mu - \beta^2} - 2$ ,

determinant of second-order principal sub form  $H(\Pi_m)_{22} = \frac{4\mu(2\mu - \beta^2)(1 - b^2) - \beta^4 b^2}{(2\mu - \beta^2)^2}$ . If the profit function has an optimal

solution, there must be  $H(\Pi_m)_{11} < 0$ ,  $H(\Pi_m)_{22} > 0$ ; that  $\mu$ ,  $\beta$  and  $b$  must meet:  $\mu b^2 < 2\mu - \beta^2$ ,

$4\mu(2\mu - \beta^2)(1 - b^2) - \beta^4 b^2 > 0$ . Profit function at this time  $\Pi_m$  is a concave function, and the optimal solution exists and is unique. Separately  $\Pi_m$  with respect to the derivatives of  $p_m$  and  $w$ ,  $p_m^{R*}$  and  $w^{R*}$  can be obtained by solving the simultaneous equations, thus theorem 2 can be obtained.

**Proof of Theorem 3**

The decision-making sequence of M-mode is that the manufacturer first sets its own wholesale price  $w$ , online direct selling price  $p_m$  and promotion effort level  $e_m$ , and the retailer then sets its own retail price  $p_r$  according to the manufacturer's decision.

The optimal solution is obtained by inverse derivation:

Firstly, the second derivative of  $\Pi_r$  to  $p_r$  is  $-2$ , which indicates that  $\Pi_r$  is a concave function with a unique maximum.

Find the first derivative of  $\Pi_r$  and make it 0 to obtain:

$$p_r = \frac{1}{2}(w + na + bp_m)$$

Bring the above formula into the manufacturer profit function  $\Pi_m$ . Can be obtained  $\Pi_m$  the Hessian determinant of  $w$ ,  $p_m$  and  $e_m$ :



$$H(\Pi_m) = \begin{vmatrix} b^2 - 2 & b & \beta \\ b & -1 & 0 \\ \beta & 0 & -\mu \end{vmatrix}$$

From the above expression, we can obtain the determinant  $h$  of the first-order principal sub form  $H(\Pi_m)_{11} = b^2 - 2 < 0$ , determinant  $h$  of second-order principal sub form  $H(\Pi_m)_{22} = 2(1 - b^2) > 0$ , determinant  $h$  of third-order principal sub form  $H(\Pi_m)_{33} = \beta^2 - 2\mu + 2\mu b^2$ , profit function  $\Pi_m$  is to have an optimal solution, then  $\mu$ ,  $\beta$  and  $b$  must be satisfied  $\beta^2 - 2\mu + 2\mu b^2 < 0$ , profit function at this time  $\Pi_m$  is a concave function with respect to  $w$ ,  $p_m$  and  $e_m$ , and there is a unique optimal solution. Separately The derivatives of  $\Pi_m$  with respect to  $w$ ,  $p_m$  and  $e_m$  can be solved by simultaneous equations to obtain  $p_m^{M^*}$ ,  $w^{M^*}$  and  $e_m^{M^*}$ , thus Theorem 3 can be obtained.